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## A Multi-Replica Decoding Technique for Contention Resolution Diversity Slotted Aloha

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*Abstract*—This paper proposes a new method for data reception over a random access channel in a satellite communication system. The method is called Multi-replicA decoding using corRelation baSed locALisAtion (MARSALA). It uses the same transmission scheme as in Contention Resolution Diversity Slotted Aloha (CRDSA) where each user sends several replicas of the same packet over the frame. MARSALA is a new decoding technique that localises all the replicas of a packet using a correlation based method, then combines them to decode the data. With MARSALA, the system can achieve a normalized throughput higher than 1.2, resulting in a significant gain compared to CRDSA, while adding a relatively low implementation complexity at the receiver. We also highlight on the practical issues related to channel estimation and how to perform coherent signal combination in MARSALA.

#### I. INTRODUCTION

Recent random access (RA) methods used in satellite communications have the potential to enhance ressource usage and reduce communication delays over a satellite return link. However, a main challenge of RA protocols is to deal with superimposed signals arriving simultaneously at the receiver. In fact, the information loss due to signals collision is the principal cause of throughput decrease in any wireless communication. To handle this problem, recent RA methods use Physical Layer Network Coding (PNC) [1] and Successive Interference Cancellation (SIC).

Among these methods we cite Contention Resolution Diversity Slotted Aloha (CRDSA) [2]. CRDSA has been proposed by the European Space Agency (ESA) and has been adopted by the DVB-RCS2 standard [3]. In CRDSA, each user sends two replicas of the same packet on random time slots on the frame. Each replica contains signalisation information used to localise its copy. If all the users have transmitted packets with equal power, only collision-free replicas can be decoded successfully at the receiver. Once decoded, both replicas are localised and removed from the frame. This operation is called interference cancellation. Thus, the receiver can recover other packets that have been in collision. This process is repeated iteratively each time that a replica is decoded successfully. In the case of equi-powered packets, CRDSA permits to achieve a higher normalized throughput ( $T_{max}\simeq 0.55)$  than the Diversity Slotted Aloha (DSA) [4] protocol in which the interference cancellation is not implemented ( $T_{max} \simeq 0.36$ ).

Another variant of CRDSA, is CRDSA++ [5], where the same concept is extended to more than two replicas per

packet (from 3 to 5 replicas). With equi-powered packets, and a Forward Error Correction (FEC) code of rate 1/2, the maximum throughput is obtained with CRDSA-3 and it is equal to 0.7. When each user transmits an irregular number of replicas on the same frame, the method is called Irregular Repetition Slotted Aloha (IRSA) [6]. If the distribution of the number of replicas is optimal, the maximum normalized throughput obtained with IRSA can reach 0.8.

Coded Slotted Aloha (CSA) [7] has been introduced later. In this method, packet segmentation and erasure coding are used in addition to PNC and SIC, but the maximum throughput achieved is equal to 0.8 which is similar to IRSA. More recently, Multi-Slot Coded Aloha [8] has been proposed to further boost the maximum achievable throughput. In MuSCA, each packet is encoded with a strong FEC code of rate R. Then the code word is divided into several fragments, and a coded localisation field is added to each fragment. The decoding entity first decodes the signalisation headers then locates all the fragments on the frame. It reassembles the fragments corresponding to one code word and attempts to decode it even if the fragments are in collision. MuSCA enables to obtain a normalized throughput greater than 1.29.

The weakness in IRSA and CSA is the limited throughput and the disadvantage of MuSCA is its implementation complexity in a real system, and the need to do important modifications to the DVB-RCS2 standard.

In this paper, we introduce Multi-replicA decoding using corRelation baSed locALisAtion (MARSALA) as a new method enabling to increase the throughput compared to CRDSA, IRSA and CSA. With MARSALA, we can decode a packet by combining all its replicas without any additionnal coding needed. MARSALA does not result in any system modifications on the CRDSA transmitter side. The only implementation complexity added is at the receiver side, and it is mainly induced by the channel estimation and the correlation based localisation. Moreover, MARSALA respects the DVB-RCS2 standard. The main contributions of our work can be summarized as follows:

- We explain precisely how packet replicas can be localised on the frame using a simple correlation based technique;
- We detail how packet replicas are combined and decoded even when they are in collision with packets sent by other users;



Fig. 1: System model

- We highlight on the channel estimation operations that should be done in order to ensure a good performance of MARSALA;
- We evaluate the normalized throughput achieved with MARSALA (i.e. the probability of successful packet transmission per time slot) and we compare it to CRDSA, in the case of equi-powered packets.

## II. SYSTEM OVERVIEW

Fig.1 shows the system model. We consider the uplink of a wireless communication system shared between  $N_u$  users. Each user transmits  $N_b$  copies of the same packet to a destination node (a satellite or a gateway) within the duration of one frame  $(T_F)$ . The frame is divided into  $N_s$  time slots. To continue to send other messages, the user must wait until the beginning of the next frame. We assume that all nodes (users and destination) operate in half duplex mode. We suppose that there is no direct link between the users. The transmission is subjected to Additive White Gaussian Noise (AWGN).

Each packet contains K bits of information. The packets are encoded with a CCSDS (Consultative Committee for Space Data Systems [9]) turbo code of rate R, wich results in a code word of length K/R bits. The code word is then modulated with a modulation of order M. The payload length obtained is equal to K/(R.log2(M)) symbols. Similar to CRDSA, a preamble and a postamble are added at the beginning and the end of each packet, and pilot blocks are distributed inside the packet for the purpose of channel estimation. The total packet length after modulation and coding is equal to L symbols. Before the transmission, the symbols corresponding to each packet enter a shaping filter with a square root raised cosine function, with an oversampling rate Q.

We suppose that the receiver memorizes  $N_s$  time slots, then lauches the CRDSA decoding process. Once CRDSA comes to a deadlock and no packet can be decoded, the receiver proceeds with MARSALA.

## III. MULTI-REPLICA DECODING USING CORRELATION BASED LOCALISATION

The proposed localisation and decoding scheme for MARSALA operates as follows:

- Localisation of  $N_b$  replicas corresponding to a same 1) packet;
- 2) For each set of  $N_b$  localised replicas:
  - Estimation and correction of the following parameters: timing offsets, clock drifts, frequency and phase offsets, signal amplitude;



Fig. 2: Example of x(t) and y(t) on a frame of 5 slots

- Combination of the  $N_b$  replicas with corrected parameters;
- Decoding of the packet.
- Interference cancellation. 3)
- 4) Come back to (1).

#### A. Replicas Localisation

The localisation entity at the receiver allows to localise all replicas corresponding to a same packet on the received frame. This operation is based on signal correlation. First, the receiver identifies a time slot containing interfered packets and refers to it by  $TS_k$ , where k is an integer index  $(k \in [1, N_s])$ . It uses the signal received within this slot as a reference signal noted by x(t), with  $t = \left| (k-1)T, (k-1)T + \frac{T}{Q}, ..., kT \right|$ , denoting the time vector, and T being the duration of one time slot. In other words, x(t) is the sum of all signals transmitted on  $TS_k$ .

Then, the receiver computes the correlation between x(t)and the signal received on the rest of the frame, noted by y(t). An example of x(t) and y(t) is given in Fig.2, where k = 2, i.e. x(t) is the signal received on the second time slot. The correlation result presents a number of correlation peaks which depends on the number of users that have transmitted a replica on  $TS_k$ . A correlation between signals on two slots is considered as a peak if the correlation amplitude is above a defined threshold. Finally, the correlation peaks are used to identify all time slots  $TS_m$ ,  $(m \in [1, N_s]; m \neq k)$ , that contain at least a packet replica of one of the packets transmitted on  $TS_k$ .

This step constitutes the main difference between MARSALA and the different versions of CRDSA. It allows to localise replicas of the same packet within the frame without the obligation of decoding the signalling information.

#### B. Channel parameters estimation and adjustment

In real transmission conditions, the replicas sent by the same user experience different channel impairments because they are sent on different time slots of the frame. In this paper, we suppose that the varying channel parameters for one user from one time slot *i* to another are the clock drift  $\Delta \tau_i$  and the phase offset  $\phi_i$ . We suppose that the timing offset  $\tau$  and the frequency offset  $\Delta f$  remain constant for one user over the duration of a frame.

Once the receiver localises the replicas of a packet on the frame, it performs signal summation (Section III-C). But, in order to ensure a coherent signal summation, the receiver has to compensate the channel impact on the corresponding signal on each time slot.

First, the receiver estimates the channel parameters relative to the localised packet replicas, that vary from one time slot to another. Therefore, it computes  $\Delta \tau_i$  and obtains the optimal sampling time for the localised replica on time slot *i*. To realize this operation, a classical algorithm for timing recovery like the algorithm of Oerder and Meyr [10] can be applied.

After the clock drift computation, the receiver estimates the phase offset  $\phi_i$ . A phase estimator like the Viterbi estimator [11] can be used. Once the estimation is done, it corrects the phase of each replica before the summation.

At the end of these operations, we obtain  $N_b$  signals corresponding to the replicas of the same packet, which are coherent in terms of timing and phase. Thus, the receiver can proceed with signal combination as explained in the following subsection.

#### C. Signal Combination

At the output of the channel estimation and adjustment entity, we obtain  $N_b$  coherent signals corresponding to  $N_b$ replicas of the same user. Each signal is interfered by different users. The receiver can therefore perform the combination of these  $N_b$  signals. The power  $P_{eq}$  of the obtained signal is

$$P_{eq} = P(s_1 + s_2 + \dots + s_{N_b}) + P(I_1 + I_2 + \dots + I_{N_b}) + \sum_{i=1}^{N_b} N_0,$$
(1)

where  $s_1, s_2, ..., s_{N_b}$  each denotes the signal relative to the replica of a same packet after phase and timing correction,  $I_1, I_2, ..., I_{N_b}$  represent the interference signals on the combined time slots and  $N_0$  is the power spectral density of AWGN. Given that the signals  $s_1, s_2, ..., s_{N_b}$  contain the same information and are coherent in phase and timing, while interference and noise are incoherent, we can write

$$P_{eq} = P(N_b s_1) + \sum_{i=1}^{N_b} P(I_i) + \sum_{i=1}^{N_b} N_0,$$
  
=  $N_b^2 P(s_1) + \sum_{i=1}^{N_b} P(I_i) + N_b N_0.$  (2)

The goal is to obtain an equivalent Signal to Noise plus Interference Ratio (SNIR) for a localised set of  $N_b$  replicas after signal combination, higher than the SNIR for one interfered replica without signal combination. Once the signals are combined, the rest of the channel parameters are estimated (Aand  $\Delta f$ ), and the receiver attempts to demodulate and decode the useful signal. If the last step is successful, the receiver performs interference cancellation, and removes the decoded packet from the corresponding time slots on the frame.

#### IV. NUMERICAL EXAMPLE

Fig.3 illustrates an example of a received frame, where  $N_s = 8$  time slots,  $N_u = 8$  users and  $N_b = 3$  replicas per packet. The modcod used in this example is QPSK 1/2 and

١.,	Slot 1	Slot 2	Slot 3	Slot 4	Slot 5	Slot 6	Slot 7	Slot 8	
4	2a	2b	<b>7</b> a	1a	3b	1b	1c	8c	
I	3a	4a		2c	5b	6c	Зc		
i		6a	 I I	4b	6b	8b	4c	ĥi	
!		<b>8</b> a	· ·	5a	7b	7c	5c	ין ו י י	

Fig. 3: Configuration of a frame with 2 clean packets,  $N_s = 8$  slots,  $N_u = 8$  users, and  $N_b = 3$  replicas per user



Fig. 4: Configuration of a frame on which MARSALA should be applied



Fig. 5: Correlation of the signal received on  $TS_1$  and the rest of the frame

all the packets are transmitted with equal power. The received packets are affected by random phase and frequency offsets. For each user, the phase offset  $\phi$  has a value between 0 and  $2\pi$  and it is supposed to change randomly from one slot to another. The frequency offset  $\Delta f$  is different for each user but it is considered to remain constant on the duration of one frame.  $\Delta f$  can have a value between 0 to 1% of the symbol rate  $1/T_s$ .

At the receiver, the frame is scanned. The packets 7a and 8c transmitted by users 7 and 8, on time slots 3 and 8 respectively, are decoded successfully because they are clean packets. Therefore, they can be removed from the frame as well as their replicas (7b, 7c) and (8a, 8b). After the interference cancellation, the new frame configuration is shown in Fig.4. All the remaining replicas are in collision. Given that all the packets are equi-powered, this situation is a deadlock for CRDSA++ and IRSA. The decoder proceeds with MARSALA in order to attempt to decode the other packets.

The receiver identifies time slot  $1 (TS_1)$  as a reference time slot. In Fig.4,  $TS_1$  contains two packets (2a) and (3a) relative to user 2 and user 3, respectively. Their copies are received on the other time slots of the frame as follows: (2b) on  $TS_2$ , (2c) on  $TS_4$ , (3b) on  $TS_5$  and (3c) on  $TS_7$ .

The objective is to localise the replicas of the packets transmitted on  $TS_1$  over the rest of the frame. Therefore, the localisation entity computes the correlation of the signal

received on  $TS_1(x(t))$  with the signal received on the rest of the frame (y(t)). The signals x(t) and y(t) can be written as

$$x(t) = s_2(t)e^{j(\phi_2 + 2\pi\Delta f_2 t)} + \underbrace{s_3(t)e^{j\phi_3 + 2\pi\Delta f_3 t} + n_1(t)}_{w(t)} \quad (3)$$

$$y(t) = s_2(t - d_2)e^{j(\phi'_2 + 2\pi\Delta f_2 t)} + s_2(t - d_4)e^{j(\phi''_2 + 2\pi\Delta f_2 t)} + \underbrace{s_3(t - d_5)e^{j(\phi''_3 + 2\pi\Delta f_3 t)} + s_3(t - d_7)e^{j(\phi''_3 + 2\pi\Delta f_3 t)} + s(t)}_{n(t)}$$
(4)

where  $s_2$  and  $s_3$  are the signals corresponding to the packets sent by user 2 and user 3 respectively.  $\phi_i$ ,  $\phi'_i$  and  $\phi''_i$  denote the phase offsets of the first, second and third replica sent by user i.  $\Delta f_i$  refers to the frequency offset and  $d_i$  represents the position, in symbol period, on the  $i^{th}$  time slot of the frame.  $n_1(t)$  is the additive white gaussian noise on TS1.

First, we focus on the signal sent by user 2. The same evaluation is done for user 3. In Eq. (5) and Eq. (6), we express x(t) and y(t) in function of the signal sent by user 2, w(t) and n(t).

$$x(t) = s_2(t)e^{j(\phi_2 + 2\pi\Delta f_2 t)} + w(t)$$
(5)

$$y(t) = s_2(t-d_2)e^{j(\phi_2'+2\pi\Delta f_2 t)} + s_2(t-d_4)e^{j(\phi_2''+2\pi\Delta f_2 t)} + n(t)$$
(6)

The correlation  $R_{Y,X}$  between x(t) and y(t) in the time domain can be written as follows

$$R_{Y,X}(\tau) = \int_{t} y(t)x^{*}(t-\tau)dt$$
  
=  $\int_{t} s_{2}(t-d_{2})e^{j(\phi_{2}'+2\pi\Delta f_{2}t)}s_{2}^{*}(t-\tau)e^{-j(\phi_{2}+2\pi\Delta f_{2}(t-\tau))}$   
+  $\int_{t} s_{2}(t-d_{4})e^{j(\phi_{2}''+2\pi\Delta f_{2}t)}s_{2}^{*}(t-\tau)e^{-j(\phi_{2}+2\pi\Delta f_{2}(t-\tau))}$   
+  $R_{n,s_{2}} + R_{n,w} + R_{s_{2},w}$  (7)

With  $R_{n,s_2} + R_{n,w} + R_{s_2,w}$  being the sum of the cross correlations between the uncorrelated terms in x(t) and y(t). For user 2, the peak amplitudes of  $R_{Y,X}(\tau)$  are obtained for  $\tau = d_2$  and  $\tau = d_4$ . The same evaluation is done for user 3. The correlation amplitudes obtained are illustrated in Fig. 5. The positions of the correlation peaks show that time slots 2, 4, 5 and 7 contain replicas of the packets present on  $TS_1$ .

In order to identify which time slots correspond to the set of replicas sent by the same user, each signal received on a time slot detected by a correlation peak, is correlated with the other time slots detected.

As explained in Section III-B, once the replicas are localised, the receiver estimates the following channel parameters for user 2: the clock drifts  $\Delta \tau_2$  and  $\Delta \tau_4$  as well as the phase offsets  $\phi_1$ ,  $\phi_2$  and  $\phi_4$  (see Fig.6). Then it performs parameters correction on the three replicas. Finally, it obtains three coherent signals to be combined and decoded. In the rest of the numerical example, we suppose that the parameters correction is perfect so the combined replicas are coherent in timing and phase.



Fig. 6: Reception of packet replicas of user 2 with a timing offset  $\tau = 0$  and a clock drift  $\Delta \tau_i$  on each time slot *i* 

Let us consider the following scenario for user 2. We suppose that the power of the interference term  $I_i$  is equal to the power of the signal sent by user  $2 (P(I_i) = P(s_2))$ . User 2 uses a Quadrature Phase Shift Keying (QPSK) modulation and a turbocode with a coding rate 1/2. The AWGN has the same power spectral density  $N_0$  on the entire frame duration. The Signal to Noise Ratio for user  $2 \left(SNR = \frac{P(s_2)}{N_0}\right)$  is supposed to be equal to 2 dB.

With the numerical assumptions considered, the SNIR for user 2 on each localised time slot i, can be written as

$$SNIR_{2,i} = \frac{1}{1 + \left(\frac{P(s_2)}{N_0}\right)^{-1}} (dB) = -2.12 \ dB.$$
(8)

Given that user 2 uses a modcod QPSK 1/2, the Packet Error Rate (PER) on each time slot *i* is equal to 1 when the SNIR is equal to -2.12 dB. After combining the 3 replicas of user 2, the equivalent SNIR becomes

$$SNIR_{eq} = \frac{N_b^2 P(s_2)}{\sum_{i=1}^3 i N_0 + i P(I_i)},$$
  
=  $\frac{3}{1 + \left(\frac{P(s_2)}{N_0}\right)^{-1}} (dB) = 2.64 \ dB.$  (9)

The PER obtained with a modcod QPSK 1/2 at 2.64 dB is lower than  $10^{-5}$ . Therefore, after signal combination, the signal sent by user 2 can be demodulated and decoded successfully.

#### V. SIMULATION RESULTS

To evaluate the performance in terms of throughput of MARSALA, we realized simulations and compared the results with the throughput obtained by CRDSA. In our simulations, we supposed that all the packets are transmitted with equal power. The channel SNR is the same for all users. To take the same assumptions as in CRDSA analysis, we considered that the channel estimation and the interference cancellation are ideal. We also assumed that the combined replicas are coherent in timing and phase.



Fig. 7: T vs G with MARSALA-3 for different values of  $E_s/N_0$ ; modcod=QPSK 1/2,  $N_s = 100$  slots

In the simulations, we still considered a frame composed of  $N_s$  time slots with  $N_u$  users attempting to transmit replicas within a frame duration. The normalized load (G) is computed as:

$$G = \frac{N_u}{N_s},\tag{10}$$

and the normalized throughput (T) is given by:

$$T = G * (1 - PLR(G))$$
, (11)

where PLR(G) is the probability that a packet is not decoded for a given G and a given SNR. To ease the recognition of several MARSALA and CRDSA versions, we denote MARSALA-2 and CRDSA-2 the MARSALA and CRDSA systems where each user transmits 2 replicas of the same packet. The same notation is taken for MARSALA-3 and CRDSA-3.

Fig.7 and Fig.8 give simulation results when the frame size is equal to 100 slots. The modcod used is QPSK 1/2. In Fig.7, the normalized throughput curves of MARSALA-3 are presented. For each studied SNR, we vary the normalized load in order to maximize the normalized throughput. We observe that, for a relatively low SNR (0 dB), MARSALA allows us to reach a throughput up to 0.7. For a higher SNR, the peak of throughput continues to increase and reaches the highest value of 1.4 at an SNR equal to 10 dB.

In Fig.8, we compare the performance in terms of normalized throughput between MARSALA and CRDSA, both with an SNR equal to 2 dB. The chosen SNR is close to the modcod ideal SNR. We can notice that the gain between the two methods is significant. The maximum normalized throughputs of MARSALA-2 and 3 are 0.8 and 1.1, respectively, while the throughput achieved by CRDSA is 1.5 times lower.

#### VI. CONCLUSION AND FUTURE WORK

In this paper, we have presented MARSALA, as a new decoding technique for CRDSA. MARSALA has proposed a solution to the deadlock problem of CRDSA, by doing simple correlation operations over the received frame, and signal summation over the time slots containing packet replicas. With the simulation assumptions considered, MARSALA has proven to achieve a higher throughput than CRDSA with a



Fig. 8: T vs G with CRDSA-2,-3 and MARSALA-2,-3; modcod=QPSK 1/2,  $E_s/N_0 = 2$  dB,  $N_s = 100$  slots

modcod QPSK 1/2. In future work, we will evaluate the effect of imperfect correction of phase and timing when the replicas of a same user are combined. Later, we will also study the gain of MARSALA with other modcods like QPSK 1/3 and higher order modulations like 8PSK and 16PSK, as well as power unbalanced packets. Furthermore, We will consider the case where MARSALA is combined to IRSA, and thus makes use of different numbers of packet replicas for each user. Then, the optimal distribution functions for the number of replicas in irregular MARSALA will be designed.

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