High Precision Satellite-based Navigation

Theory and Applications

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Thanks to our team and may more!

Multi Sensor Systems Group

- **Daniel Medina** – Group leader, GNSS & *robust estimation*
- **Christoph Lass** – deputy leader, *Precise GNSS*
- **Andrea Bellés** – *Precise GNSS & Machine Learning*
- **Hakan Uyanik** – *GNSS Jamming Detection*
- **Iulian Filip** – *LiDAR and Visual SLAM*
- **Alonso Llorente** – *SLAM and DigitalTwin simulations*
- **Filippo Rizzi** – *Multi-Sensor Architectures for PNT*

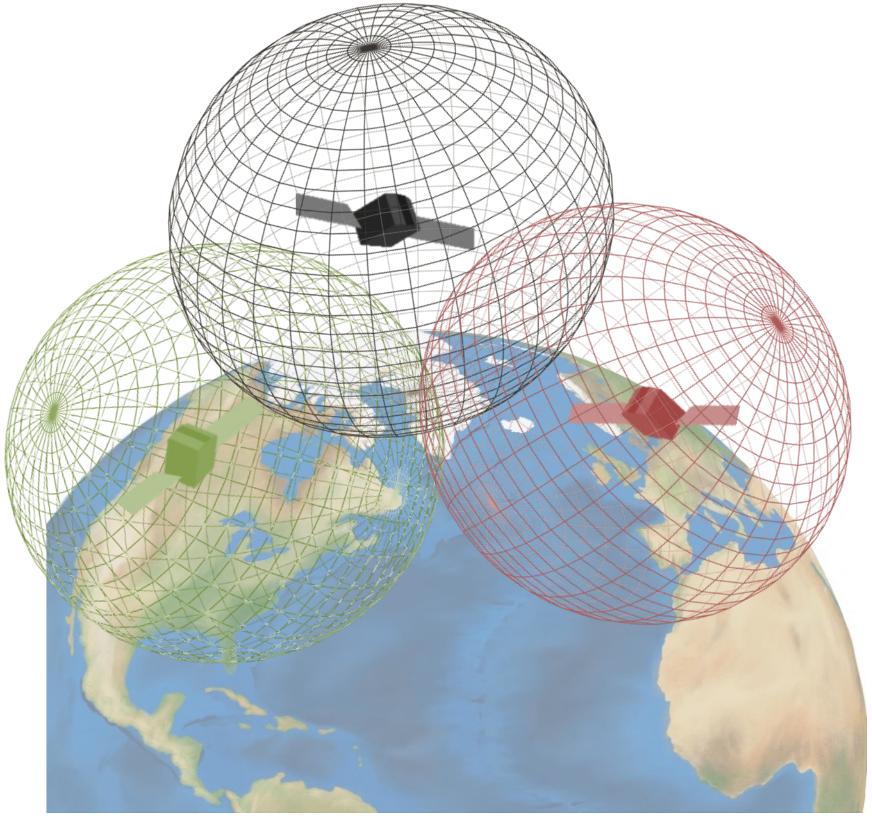
Further collaborators

- **My PhD advisor: Jordi Vilà-Valls**
- **My dear French collaborators**: Éric Chaumette, Lorenzo Ortega, Paul Chauchat, Samy Labsir, François Vincent
- **Northeastern University:** Pau Closas, Haoqing Li, Helena Calatrava
- Spanish colleagues: Juan Manuel Gandarias, Gonzalo Seco
- **Former colleagues:** Anja, Xiangdong, JuanMar, Lukas, Iván, etc.

Thank you very much TéSA, Lorenzo and Julien for bringing me here!

“GNSS are everywhere”

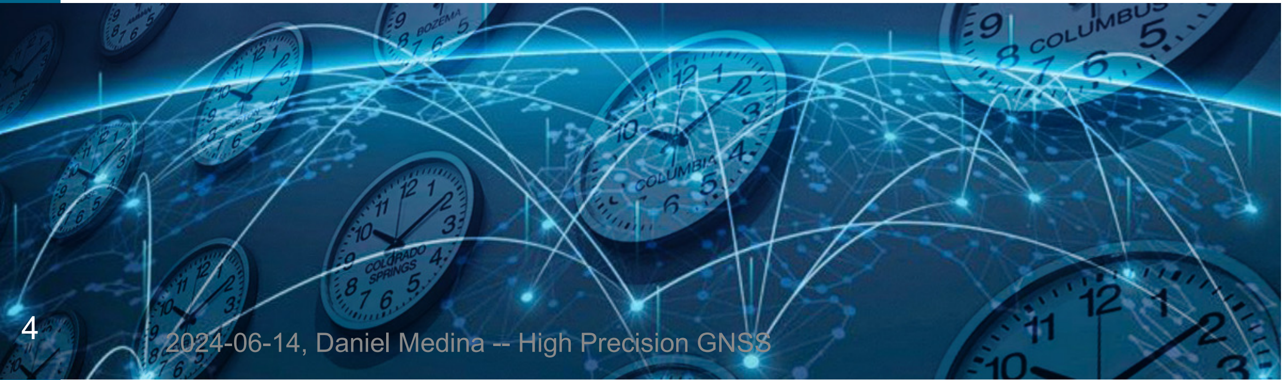
Global Navigation Satellite Systems (GNSS) have become the cornerstone for worldwide localization and timing



“GNSS are everywhere”

GNSS presence extends across financial, energy grid, mass-market or vehicular applications

Prospective autonomous systems pose stringent navigation requirements



Overview on GNSS

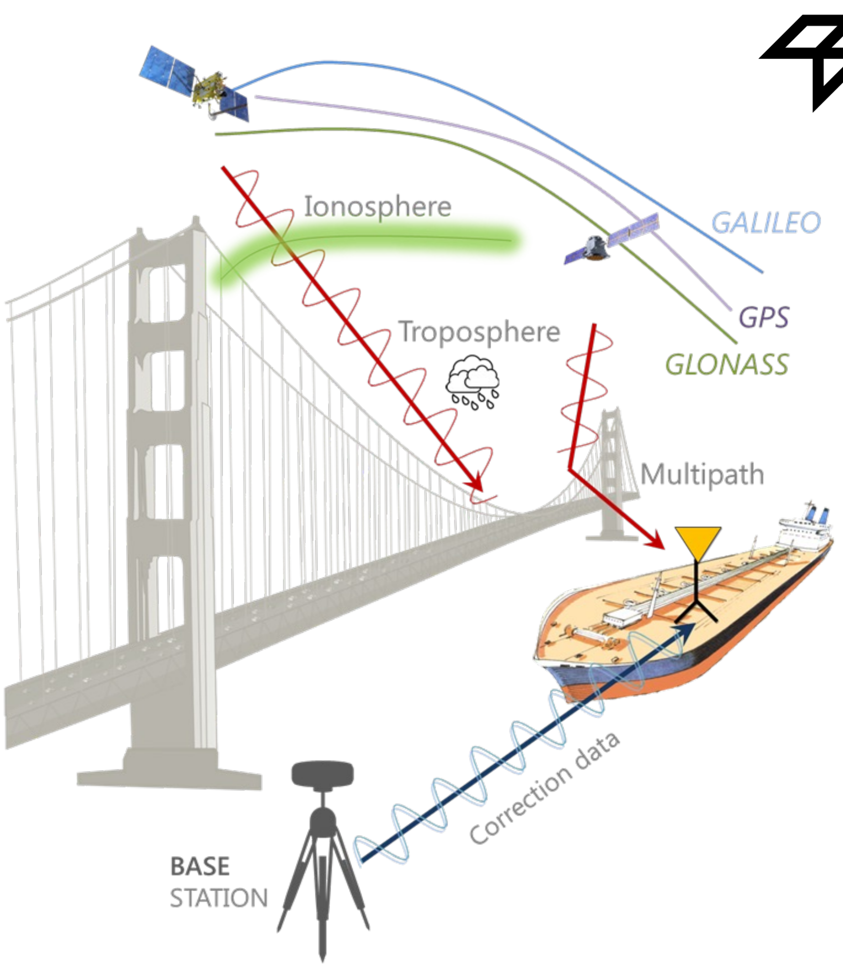
GNSS is the main information source for Positioning, Navigation and Timing (PNT)

Challenge #1: Precision

The accuracy of standard *code*-based navigation is limited → <10 meters positioning & poor attitude

Challenge #2: Robustness

Multipath and other local effects can severely degrade the performance → large errors



- The use of carrier phase observations is the key for high precision navigation!

Motivation for Precise GNSS

Intelligent vehicles

- For people: autonomous cars, assisted landing, etc.
- For services: package delivery, photogrammetry, farming, etc.

Aerospace

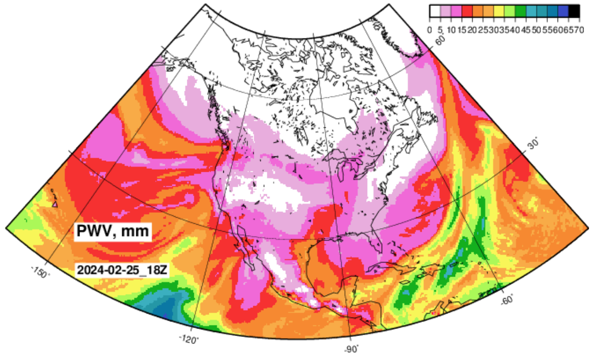
- Spacecraft orientation
- Satellite orbit determination

Natural sciences

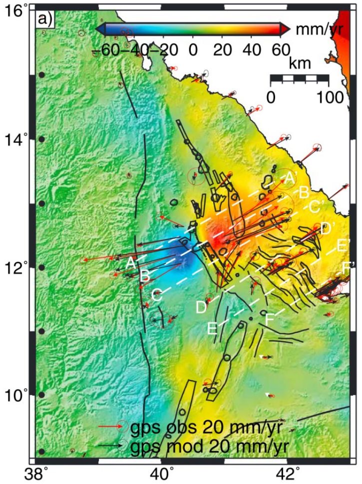
- Meteorology
- Crustal movement
- Solar terrestrial physics

[Demand For These Autonomous Delivery Robots Is Skyrocketing During This Pandemic (forbes.com)](forbes.com)

GPS installation and use on the IGS (Gomez, ION GNSS 2004)



NOAA Precipitable Water Vapor (PWV) forecast



NOAA Precipitable Water Vapor (PWV) forecast

In today's seminar…

- **Basics of carrier phase measurements**: how to get them? How to use them?
- **High precision GNSS techniques**: PPP, RTK and the new horizon
- **Multi antenna systems**: orientation and navigation with carrier phase
- **Cooperative GNSS:** exchanging information in a network for better navigation
- **Research & industry perspectives**

Outline

1 What are Carrier Phase Observations?
  - Carrier Phase Limitations

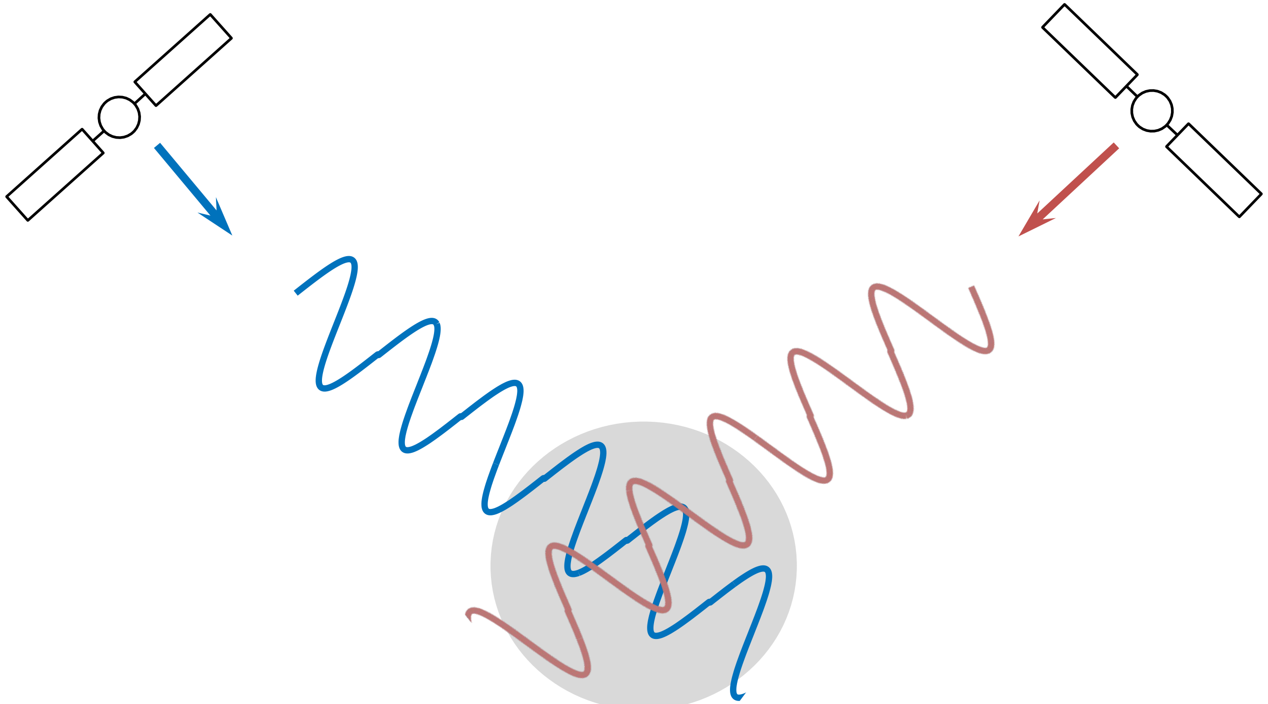
2 Precise Positioning Techniques
  - Real Time Kinematics (RTK)
  - Precise Point Positioning (PPP)

3 Multi-Antenna Systems

4 Cooperative GNSS Positioning

Precise Positioning

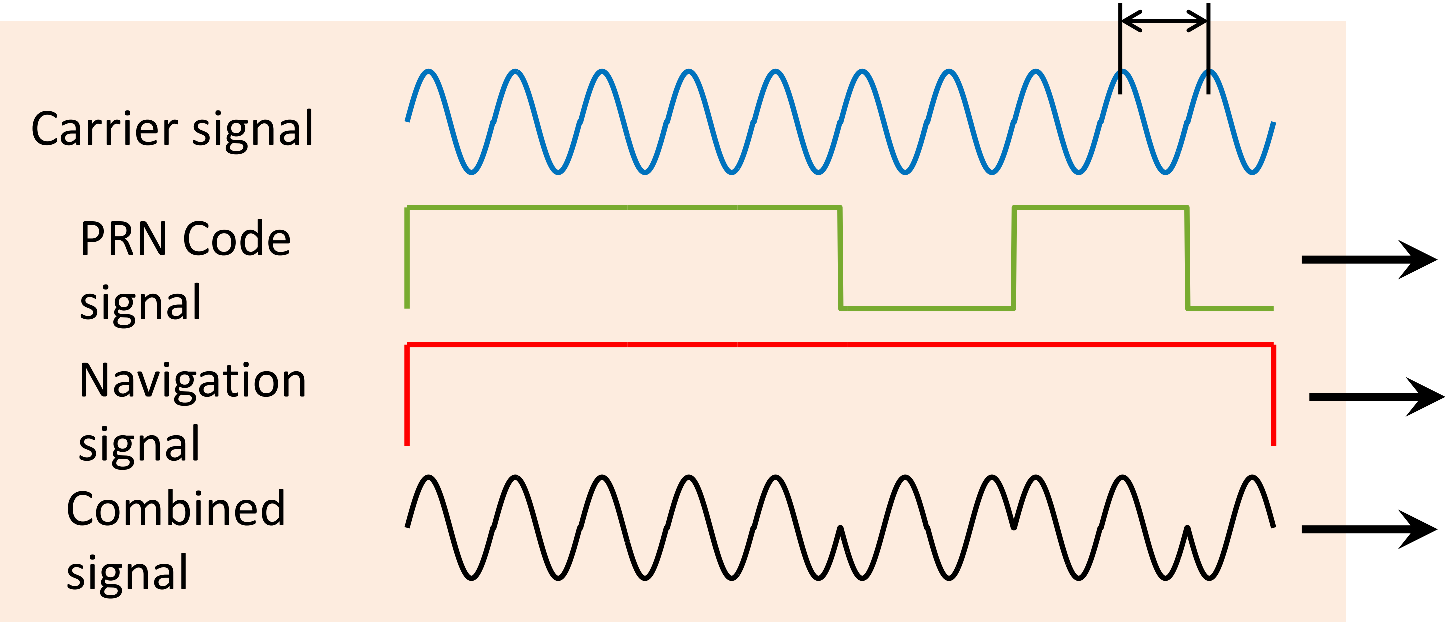
Receiver to positioning performance



What are carrier phase observations?

GNSS signal structure

$\lambda \equiv \text{Wavelength}$



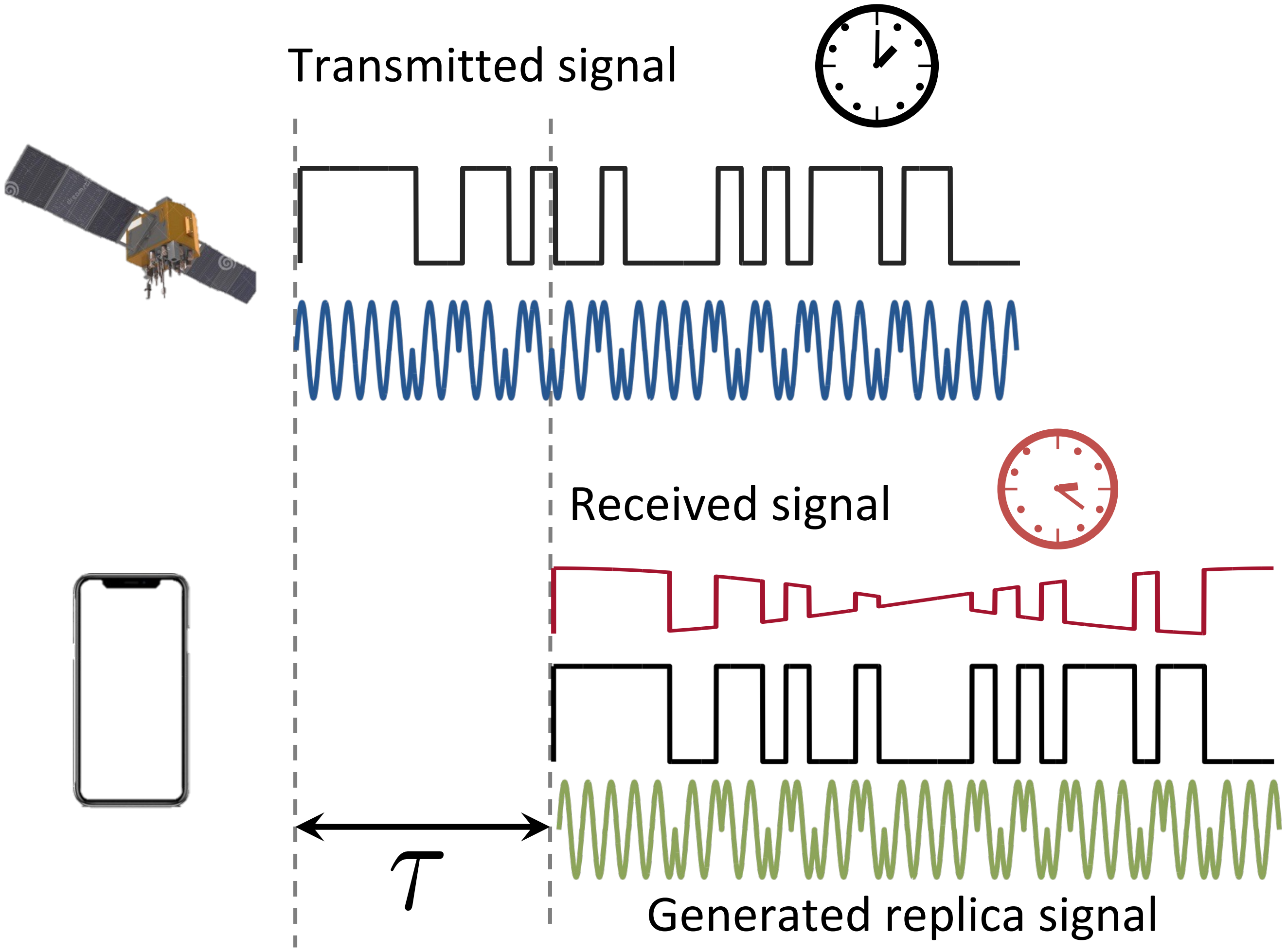
The frequency of the GNSS determines the wavelength $\lambda = \frac{c}{f}$

Which satellite is talking to me?

Where is the satellite?

An atenuated (and distorded) version of this signal is what we actually receive

What are carrier phase observations?



$$\tau \equiv \text{time between transmission and reception}$$

Code observations

- Derived from the apparent time of flight

Carrier phase observations

- Derived from the carrier phase offset
- Contains a fractional + integer part

Code and carrier phase observation models

Code observation

$$\rho^i = \|\boldsymbol{p}^i - \boldsymbol{p}\| + I^i + T^i + c\left(dt - dt^i\right) + \varepsilon^i$$

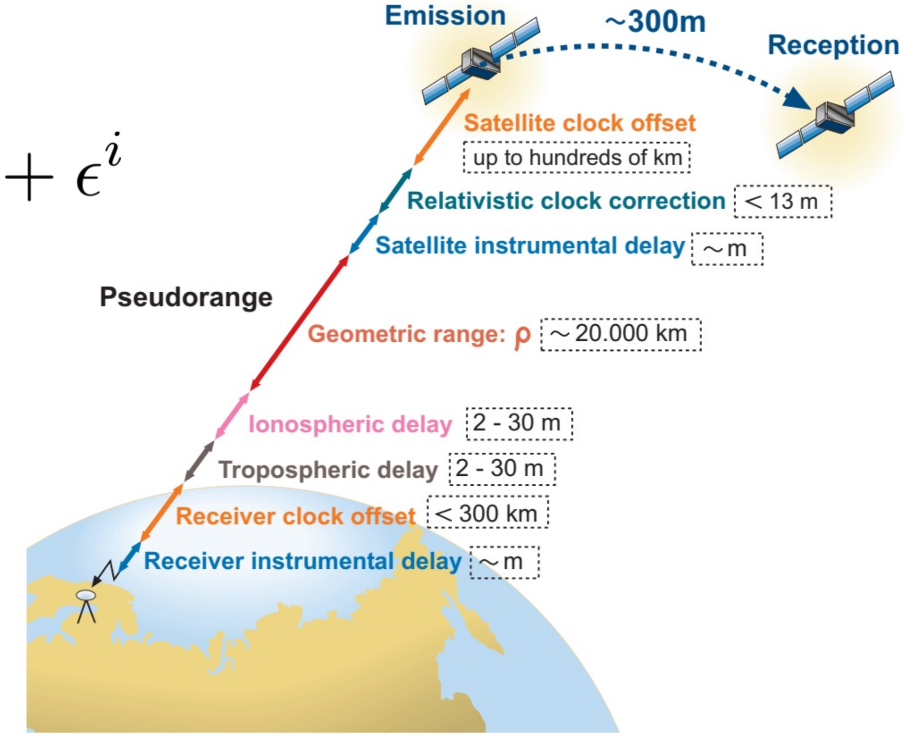
Carrier phase observation

$$\Phi^i = \|\boldsymbol{p}^i - \boldsymbol{p}\| - I^i + T^i + c\left(dt - dt^i\right) + \lambda N^i + \epsilon^i$$

Code and carrier phase observations look very similar

- Carrier phase do not add any additional positioning info
- Carrier phase are even a bit more complicated

So… why to use carrier phase??



Sanz Subirana, J., J. M. Juan Zornoza, and M. Hernández-Pajares. "GNSS Data Processing, Volume I: Fundamentals and Algorithms." *ESA Communications, ESTEC, Noordwijk, Netherlands* (2013)

Code and carrier phase observation models

Code observation

$$\rho^i = \|\boldsymbol{p}^i - \boldsymbol{p}\| + I^i + T^i + c\left(dt - dt^i\right) + \varepsilon^i$$

Carrier phase observation

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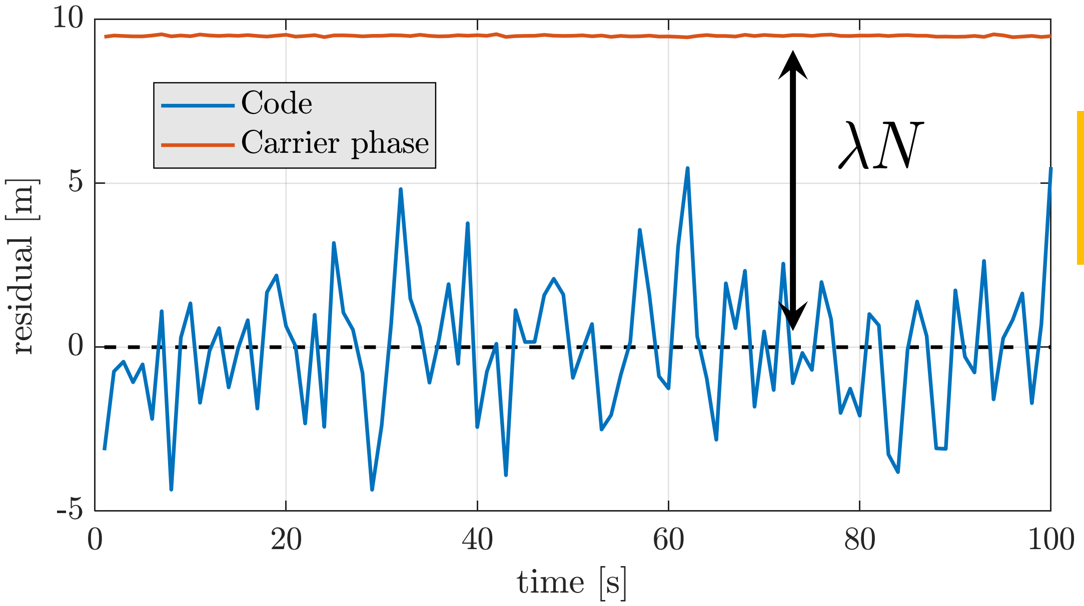
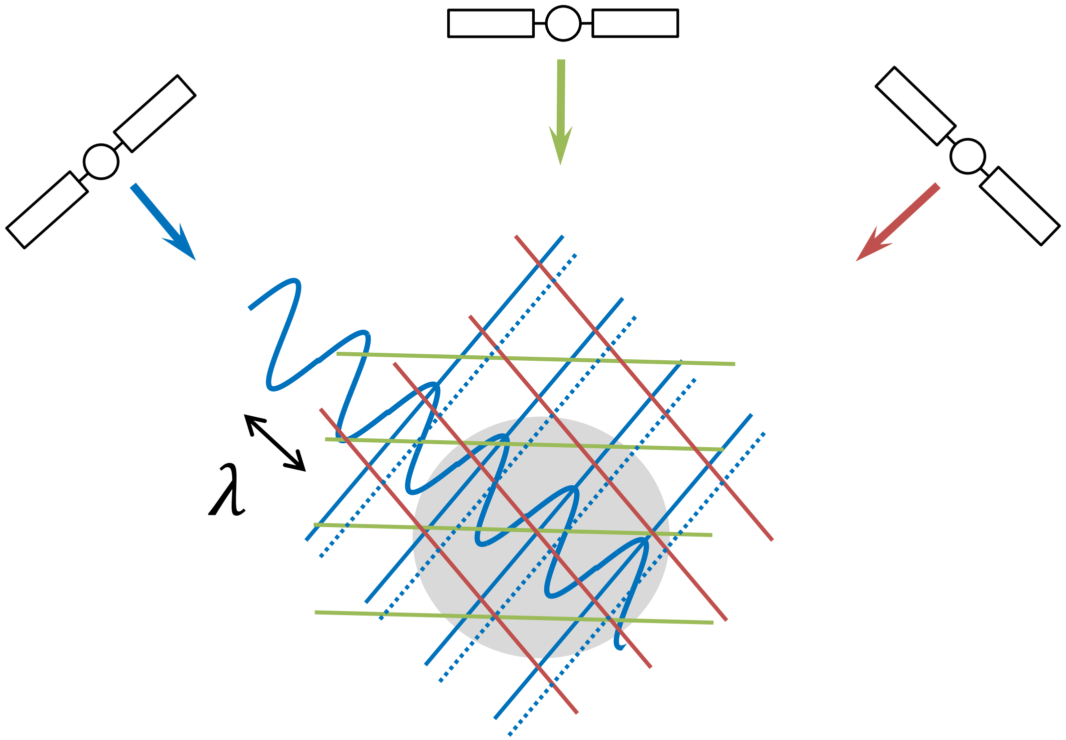


Noise on carrier phase observations is two orders of magnitude lower than code noise

$$\varepsilon \sim \mathcal{N}\left(0, \sigma_\rho^2\right), \ \sigma_\rho \simeq 2-5 \ [\mathrm{m}]$$

$$\epsilon \sim \mathcal{N}\left(0, \sigma_\Phi^2\right), \ \sigma_\Phi \simeq 2 \ [\mathrm{mm}]$$

Working principle for carrier phase-based positioning



Recap on standard (code-based) positioning

Positioning methods

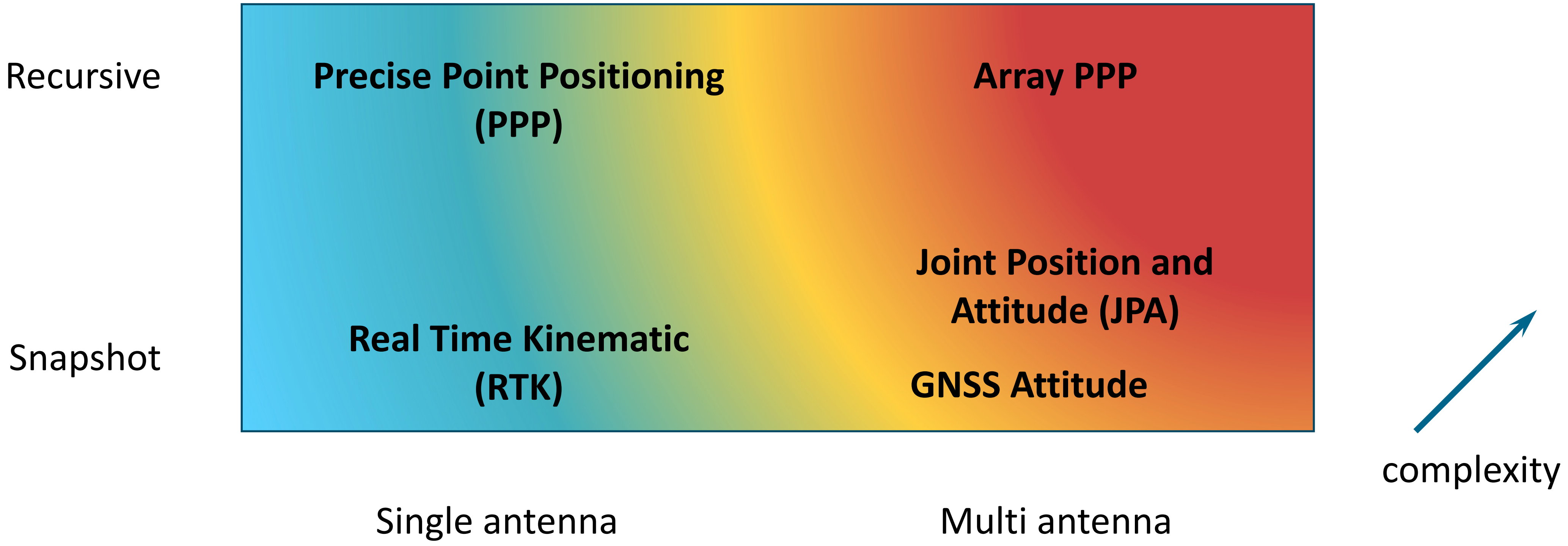
- Single Point Positioning (SPP)
- Differential and augmented GNSS (DGNSS)

Estimation methods

- Snapshot / memoryless: Maximum Likelihood Estimation (Least Squares)
- Recursive: Maximum A Posteriori (Kalman Filter)

GNSS high precision is all about carrier phase…
+ correction data + integer estimation

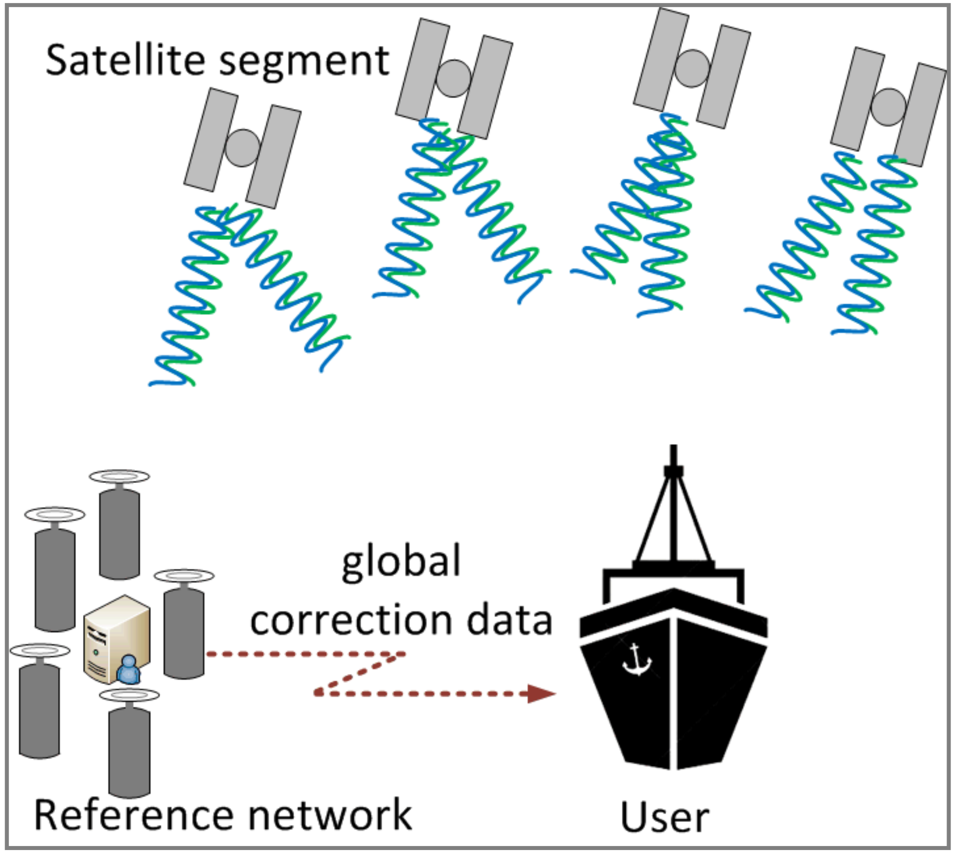
High Precision GNSS Techniques



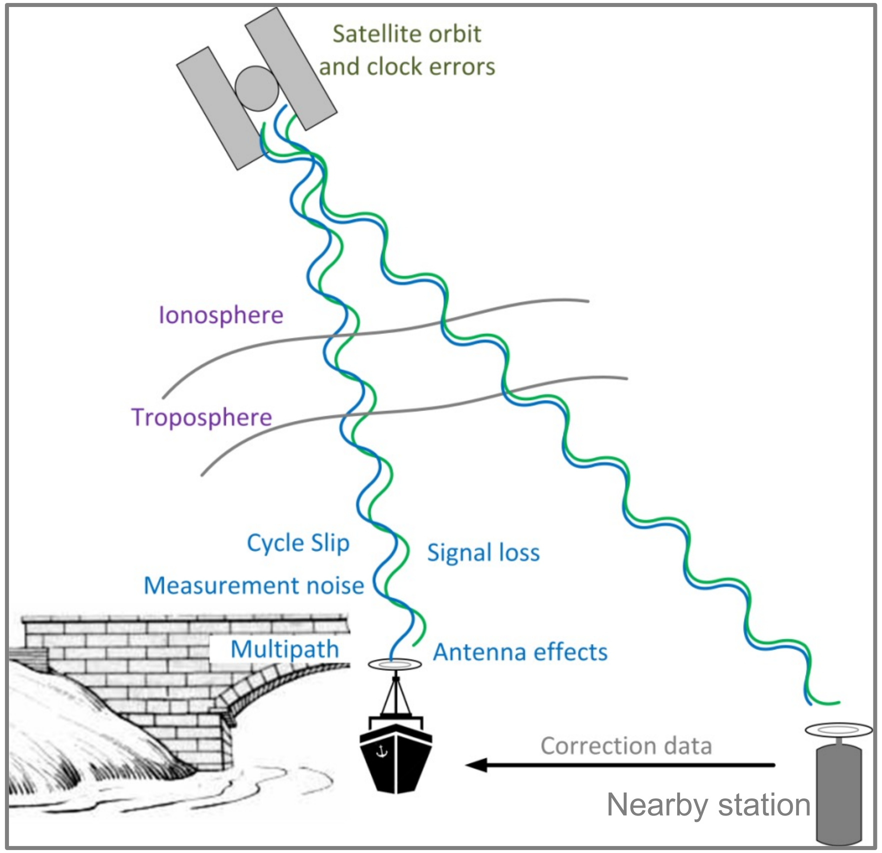
	Single antenna	Multi antenna
Recursive	**Precise Point Positioning (PPP)**	**Array PPP**
Snapshot	**Real Time Kinematic (RTK)**	**Joint Position and Attitude (JPA)** **GNSS Attitude**

High precision GNSS techniques

Precise **P**oint **P**ositioning (**PPP**)

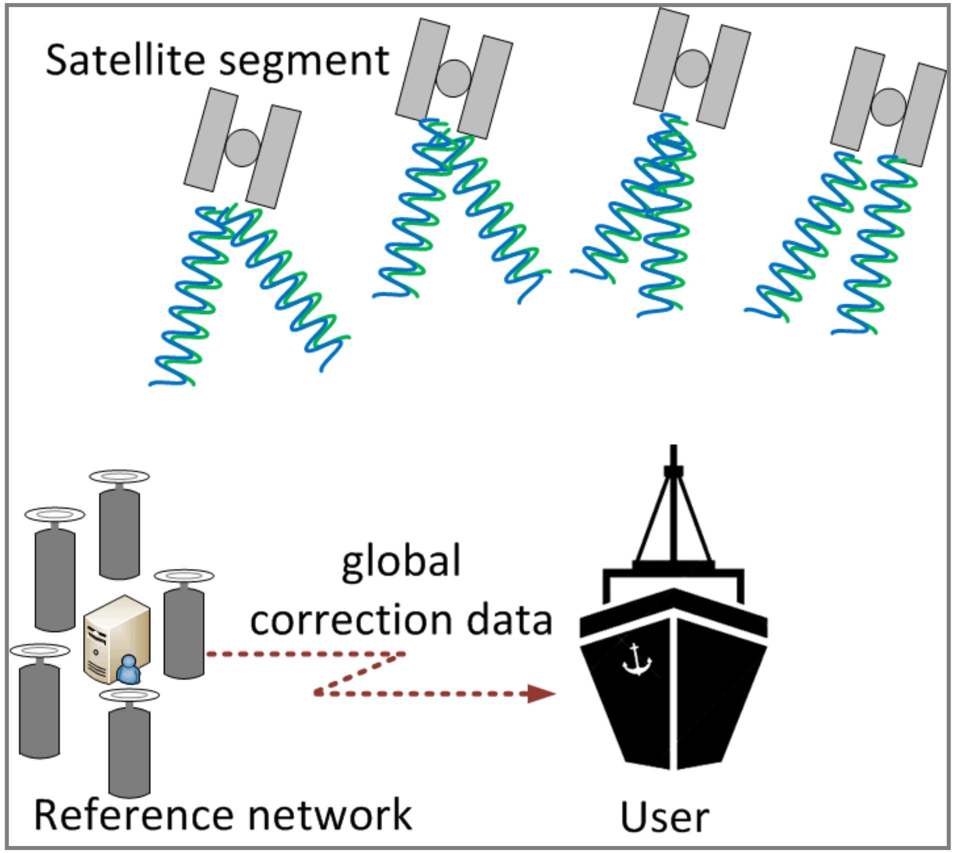


Real **T**ime **K**inematic (**RTK**)



Precise Point Positioning

Precise **P**oint **P**ositioning (**PPP**)

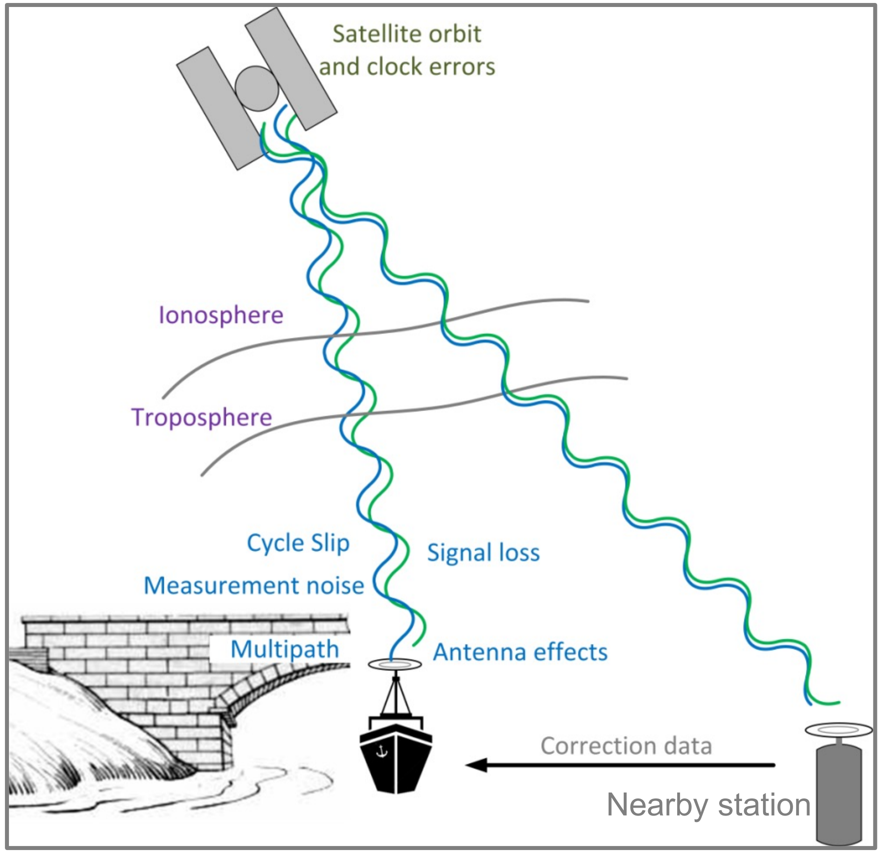


- ✓ "absolute" positioning, **no need for a nearby reference station**
- Global network of ground stations necessary → corrections on satellite orbits, clock, biases, atmospheric delays, etc.
- Precise ephemeris cannot be deployed in real time (* Galileo High Accuracy Service (HAS), Kepler)
- Several unknowns to be estimated: clock offsets, troposheric delays, ambiguities, …
- **Estimation process:** recursive with Kalman Filter
- **Challenge:** convergence time, quality of corrections
- **Accuracy:** decimeter up to centimeter

Real Time Kinematic

- **RTK is a differential phase-based positioning** → base station of known coordinates transmits its data
- The estimation technique is well studied (but still challenging!)
- **Estimation process:** Kalman Filtering or Least Squares
- **Challenge: need for nearby stations + communication, computation complexity**
- **Accuracy: (instantaneous) centimeter to millimeter**

Real Time Kinematic (RTK)



RTK Processing

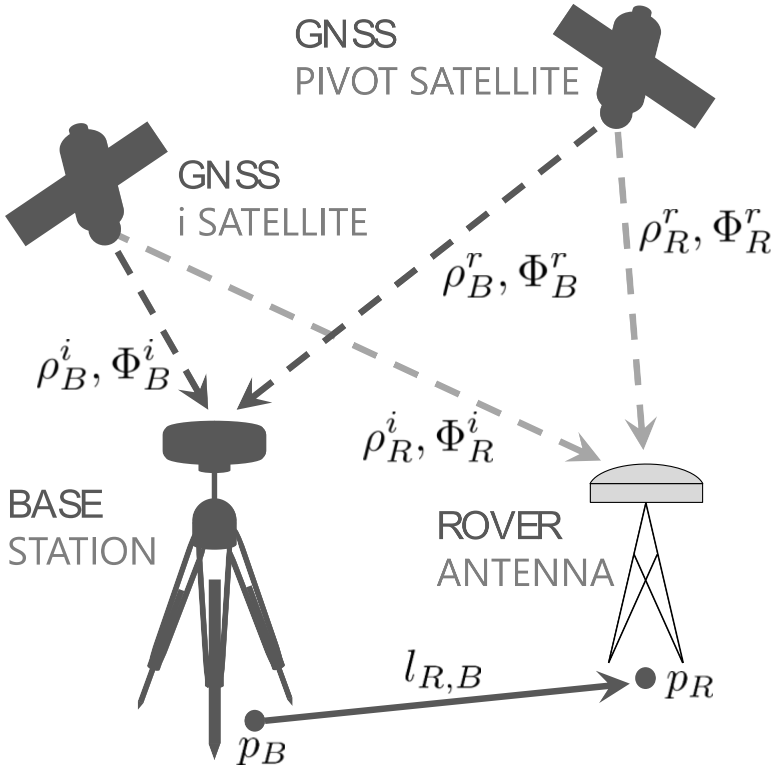
$$
\begin{aligned}
&\Phi_B^i = \|\boldsymbol{p}^i - \boldsymbol{p}_B\| - I^i + T^i + c\left(-dt^i + dt_B\right) + \lambda N_B^i + \epsilon_B^i \\
(-)\quad &\Phi_R^i = \|\boldsymbol{p}^i - \boldsymbol{p}_R\| - I^i + T^i + c\left(-dt^i + dt_R\right) + \lambda N_R^i + \epsilon_R^i \\
\\
&\Phi_B^r = \|\boldsymbol{p}^r - \boldsymbol{p}_B\| - I^r + T^r + c\left(-dt^r + dt_B\right) + \lambda N_B^r + \epsilon_B^r \\
(-)\quad &\Phi_R^r = \|\boldsymbol{p}^r - \boldsymbol{p}_R\| - I^r + T^r + c\left(-dt^r + dt_R\right) + \lambda N_R^r + \epsilon_R^r
\end{aligned}
$$

Single-differencing → removes ionospheric and tropospheric effects

Double-differencing → eliminates the clock offsets and satellite biases

$$DD(\cdot)^i \equiv (\cdot)_{R,B}^{i,r} = (\cdot)_R^i - (\cdot)_B^i - \left((\cdot)_R^r - (\cdot)_B^r\right)$$

$$(\cdot) = \{\Phi, \rho\}$$



RTK Processing

State estimate

$$\mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}, \ \mathbf{a} \in \mathbb{Z}^n, \ \mathbf{b} \in \mathbb{R}^3$$

Set of observations

$$\mathbf{y} = \begin{bmatrix} \boldsymbol{DD\Phi} \\ \boldsymbol{DD\rho} \end{bmatrix}, \ \mathbf{y} \in \mathbb{R}^{2n}$$

Observation model

$$DD\Phi^i = -\left(\mathbf{u}^i - \mathbf{u}^r\right)^\top \mathbf{b} + \lambda a^i + \epsilon_{R,B}^{i,r}$$

$$DD\rho^i = -\left(\mathbf{u}^i - \mathbf{u}^r\right)^\top \mathbf{b} + \varepsilon_{R,B}^{i,r}$$

$$\mathbb{E}(\mathbf{y}) = \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{b}$$

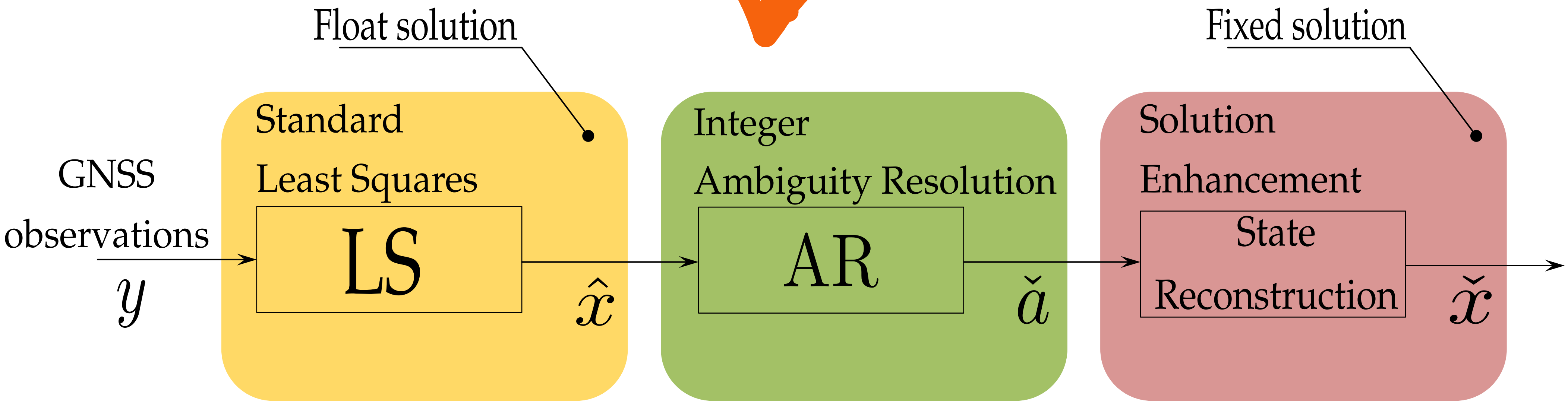
$$\mathbb{D}(\mathbf{y}) = \mathbf{Q}_{\mathbf{y}}$$

☆! Also known as Mixed Estimation Model

RTK / mixed model estimation

$$\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = \arg \min_{\mathbf{a}\in\mathbb{Z}^n,\mathbf{b}\in\mathbb{R}^3} \|\mathbf{y}-\mathbf{A}\mathbf{a}-\mathbf{B}\mathbf{b}\|^2_{\mathbf{Q}_\mathbf{y}}$$

A 3-step decomposition is applied



Float solution

Fixed solution

GNSS observations y

Standard Least Squares

LS

$\hat{x}$

Integer Ambiguity Resolution

AR

$\check{a}$

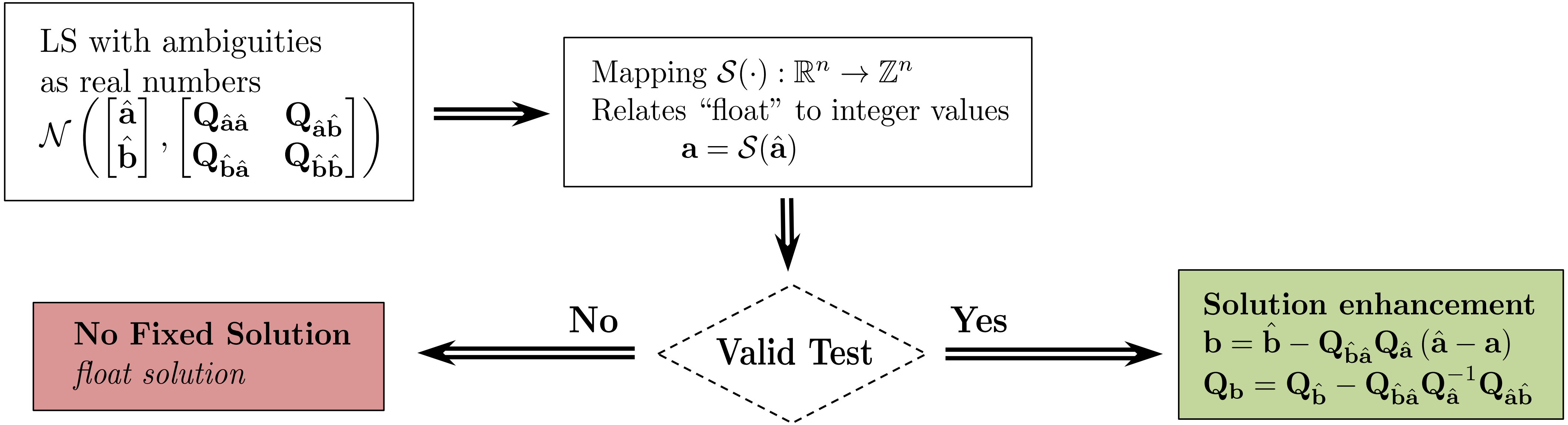
Solution Enhancement

State Reconstruction

$\check{x}$

RTK / mixed model estimation

$$\underbrace{\min_{\substack{\hat{\mathbf{a}}\in\mathbb{R}^n\\ \hat{\mathbf{b}}\in\mathbb{R}^3}} \|\mathbf{y}-\mathbf{A}\hat{\mathbf{a}}-\mathbf{B}\hat{\mathbf{b}}\|^2_{\mathbf{Q}_{\mathbf{y}}}}_{\textbf{Float solution}} + \underbrace{\min_{\mathbf{a}\in\mathbb{Z}^n} \|\hat{\mathbf{a}}-\mathbf{a}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}}}}_{\textbf{Integer Ambiguity Resolution}} + \underbrace{\min_{\mathbf{b}\in\mathbb{R}^3} \|\hat{\mathbf{b}}|\mathbf{a}-\mathbf{b}\|^2_{\mathbf{Q}_{\hat{\mathbf{b}}|\mathbf{a}}}}_{\textbf{Fixed solution}}$$



LS with ambiguities
as real numbers

$$\mathcal{N}\left(\begin{bmatrix}\hat{\mathbf{a}}\\ \hat{\mathbf{b}}\end{bmatrix}, \begin{bmatrix}\mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{b}}}\\ \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} & \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{b}}}\end{bmatrix}\right)$$

Mapping $\mathcal{S}(\cdot) : \mathbb{R}^n \to \mathbb{Z}^n$
Relates "float" to integer values
$\mathbf{a} = \mathcal{S}(\hat{\mathbf{a}})$

Valid Test

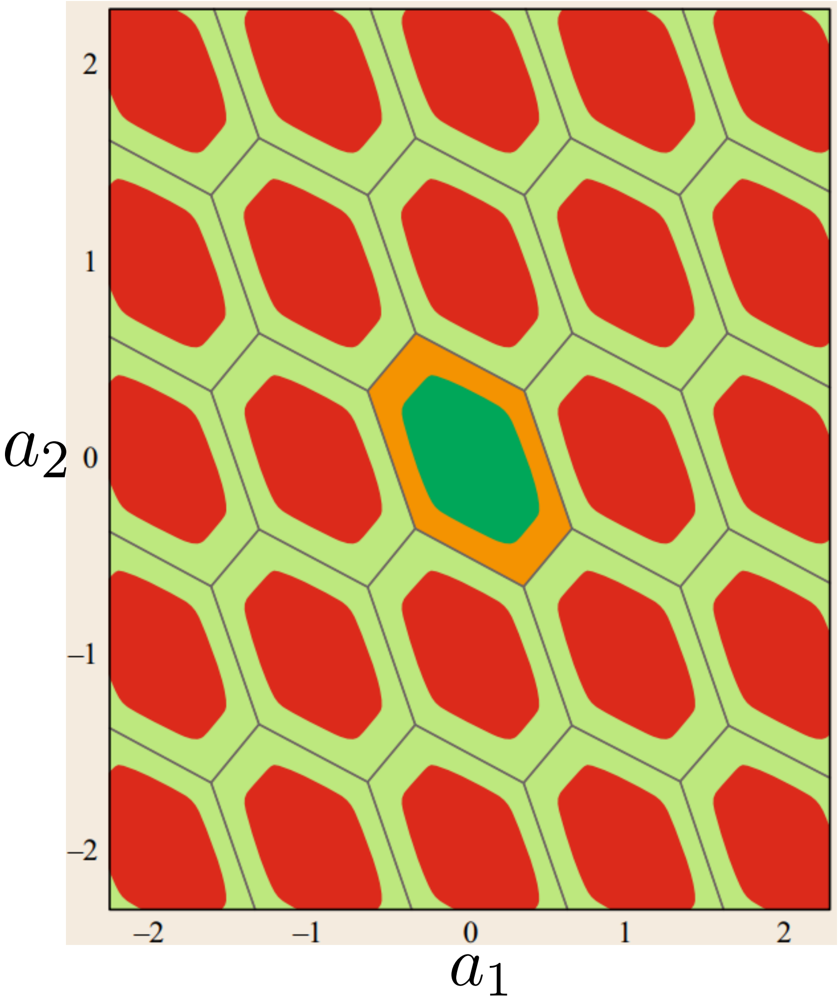
No

No Fixed Solution
float solution

Yes

Solution enhancement
$\mathbf{b} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Q}_{\hat{\mathbf{a}}}\left(\hat{\mathbf{a}} - \mathbf{a}\right)$
$\mathbf{Q}_{\mathbf{b}} = \mathbf{Q}_{\hat{\mathbf{b}}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Q}_{\hat{\mathbf{a}}}^{-1}\mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{b}}}$

Integer Ambiguity Resolution (IAR)



IAR is the theoretical framework for integer estimation + hypothesis testing on their reliability

- It is an *n*-hiperdimensional ellipsoidal search
- The success of the process depends on:
  - Quality of the observation model
  - Number of observations

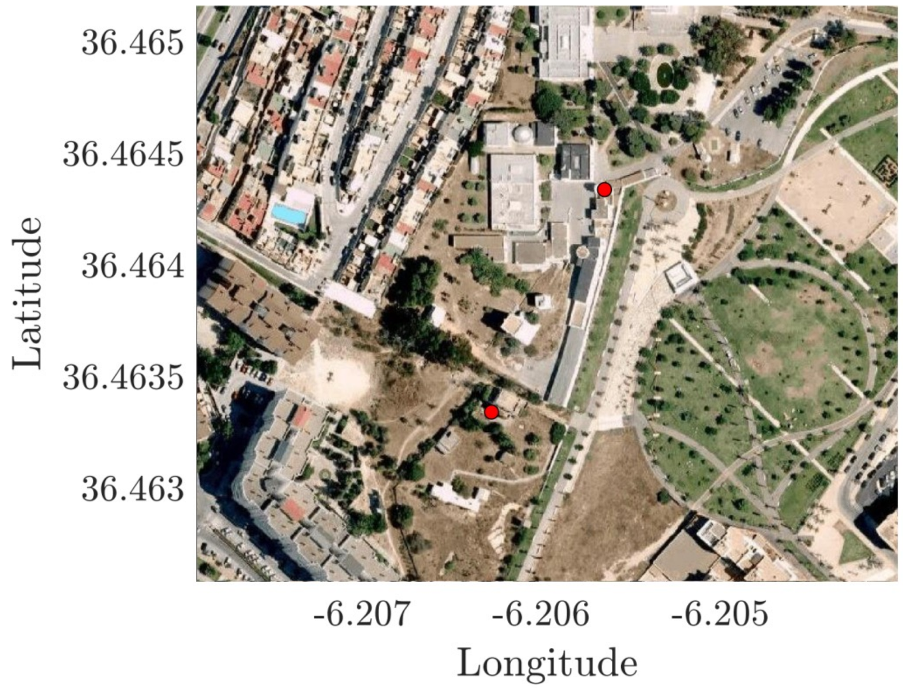
$$\mathcal{S}(\hat{\mathbf{a}}) = \begin{cases} \check{\mathbf{a}} & \text{if } \hat{\mathbf{a}} \in \Omega_a \quad \text{(success)} \\ \check{\mathbf{a}} \neq \mathbf{a} & \text{if } \hat{\mathbf{a}} \in \Omega / \Omega_a \quad \text{(failure)} \\ \hat{\mathbf{a}} & \text{if } \hat{\mathbf{a}} \in \Omega \quad \text{(undecided)} \end{cases}$$

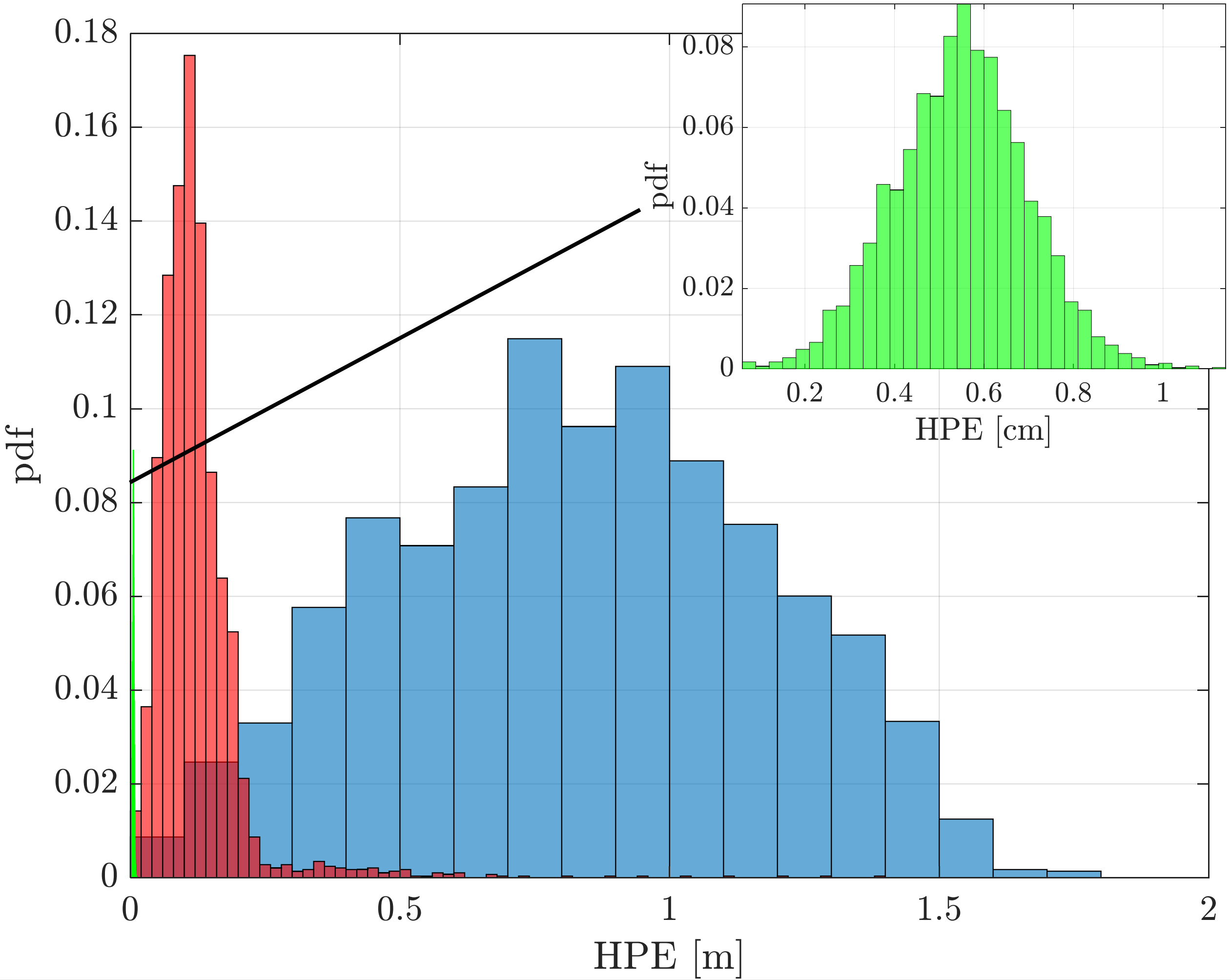
Teunissen, Peter JG., and Verhagen S. "Integer Aperture Estimation." *Inside GNSS* (2011).
Teunissen, Peter, and Oliver Montenbruck, eds. *Springer handbook of global navigation satellite systems*. Springer, 2017.

Putting all together: RTK in action!

San Fernando IGS stations

2019, DOY 001, 00:00 – 23:59



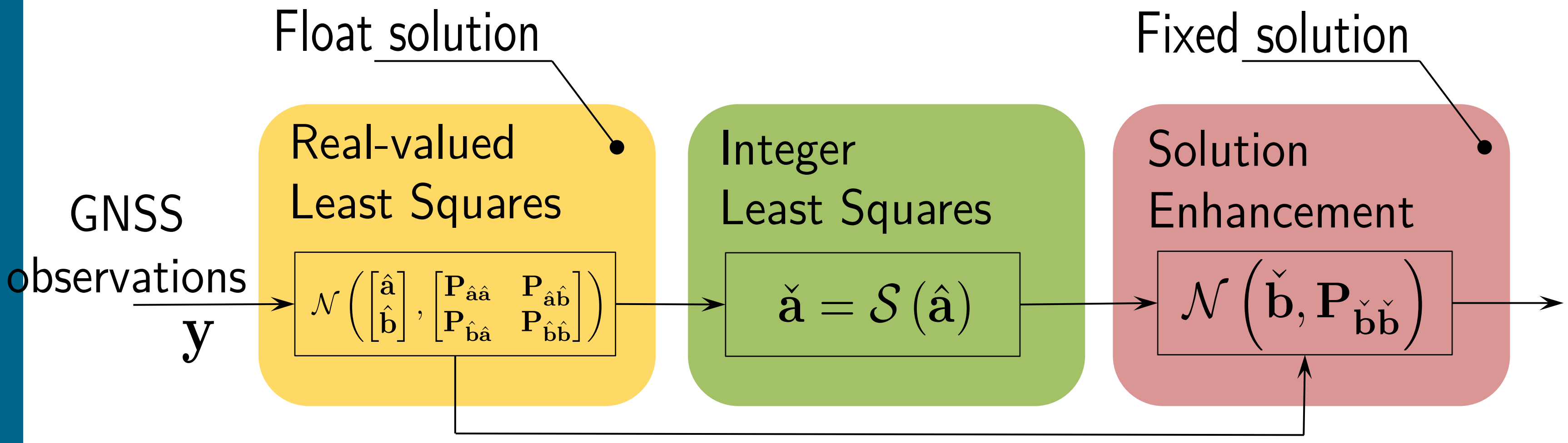


Castro-Arvizu, J. M., Medina, D., Ziebold, R., Vilà-Valls, J., Chaumette, E., & Closas, P. (2021). Precision-aided partial ambiguity resolution scheme for instantaneous RTK positioning. Remote sensing, 13(15), 2904.

Further on the Mixed Model Estimation

The RTK problem can be cast as a minimization over integer- and real-valued parameters:

$$\min_{\substack{\mathbf{a}\in\mathbb{Z}^n\\ \mathbf{b}\in\ ^3}} \|\mathbf{y}-\mathbf{A}\mathbf{a}-\mathbf{B}\mathbf{b}\|^2_{\boldsymbol{\Sigma}} \rightarrow \underbrace{\left\|\mathbf{y}-\mathbf{A}\hat{\mathbf{a}}-\mathbf{B}\hat{\mathbf{b}}\right\|^2_{\boldsymbol{\Sigma}}}_{\text{Float solution}} + \min_{\mathbf{a}\in\mathbb{Z}^n}\left(\underbrace{\|\hat{\mathbf{a}}-\mathbf{a}\|^2_{\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}}_{\text{Integer solution}} + \underbrace{\min_{\mathbf{b}\in\mathbb{R}^3}\left\|\hat{\mathbf{b}}(\mathbf{a})-\mathbf{b}\right\|^2_{\mathbf{P}_{\hat{\mathbf{b}}(\mathbf{a})}}}_{\text{Fixed solution}}\right)$$



Float solution

Fixed solution

GNSS observations $\mathbf{y}$

Real-valued Least Squares

$\mathcal{N}\left(\begin{bmatrix}\hat{\mathbf{a}}\\ \hat{\mathbf{b}}\end{bmatrix}, \begin{bmatrix}\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} & \mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{b}}}\\ \mathbf{P}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} & \mathbf{P}_{\hat{\mathbf{b}}\hat{\mathbf{b}}}\end{bmatrix}\right)$

Integer Least Squares

$\check{\mathbf{a}} = \mathcal{S}\left(\hat{\mathbf{a}}\right)$

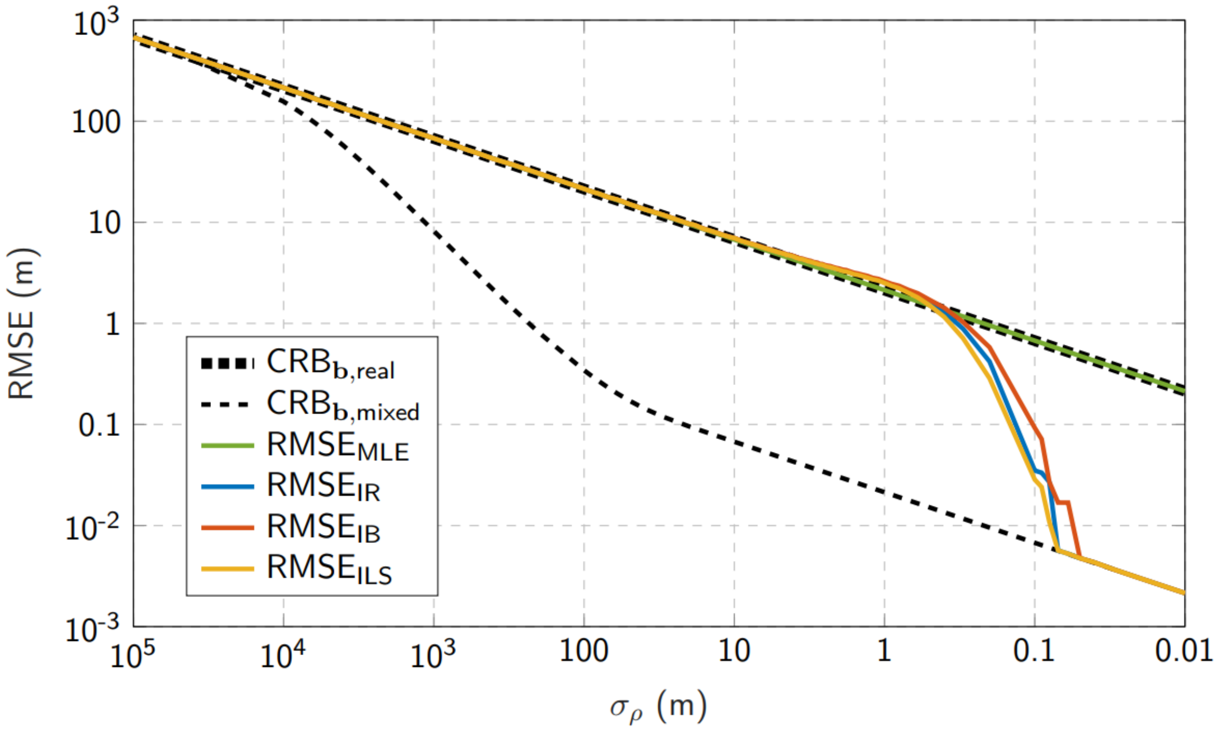
Solution Enhancement

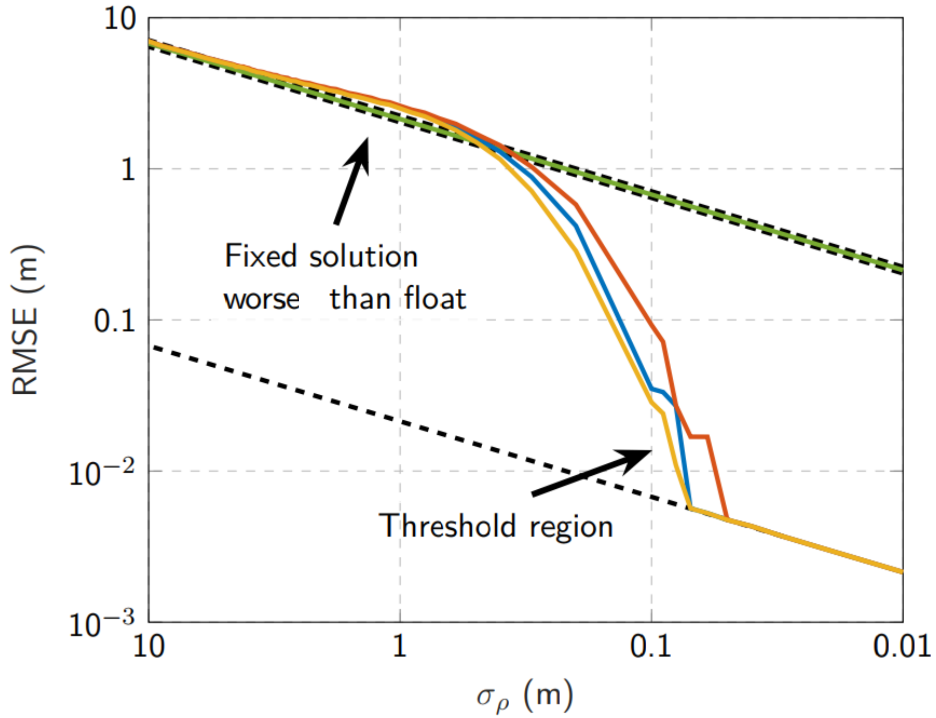
$\mathcal{N}\left(\check{\mathbf{b}}, \mathbf{P}_{\check{\mathbf{b}}\check{\mathbf{b}}}\right)$

- Are these estimators efficient?
- What is the best achievable performance?

Cramér Rao Bound for the Mixed Estimation Problem

- **Estimation bounds for the real/integer problem?** Not available!
  - Proposed Cramér-Rao lower bound (CRB) for the mixed model estimation
- **Is this three-step estimation procedure efficient?**
  - Three methods: integer rounding (IR), integer bootstrapping (IB) and integer least squares (ILS)

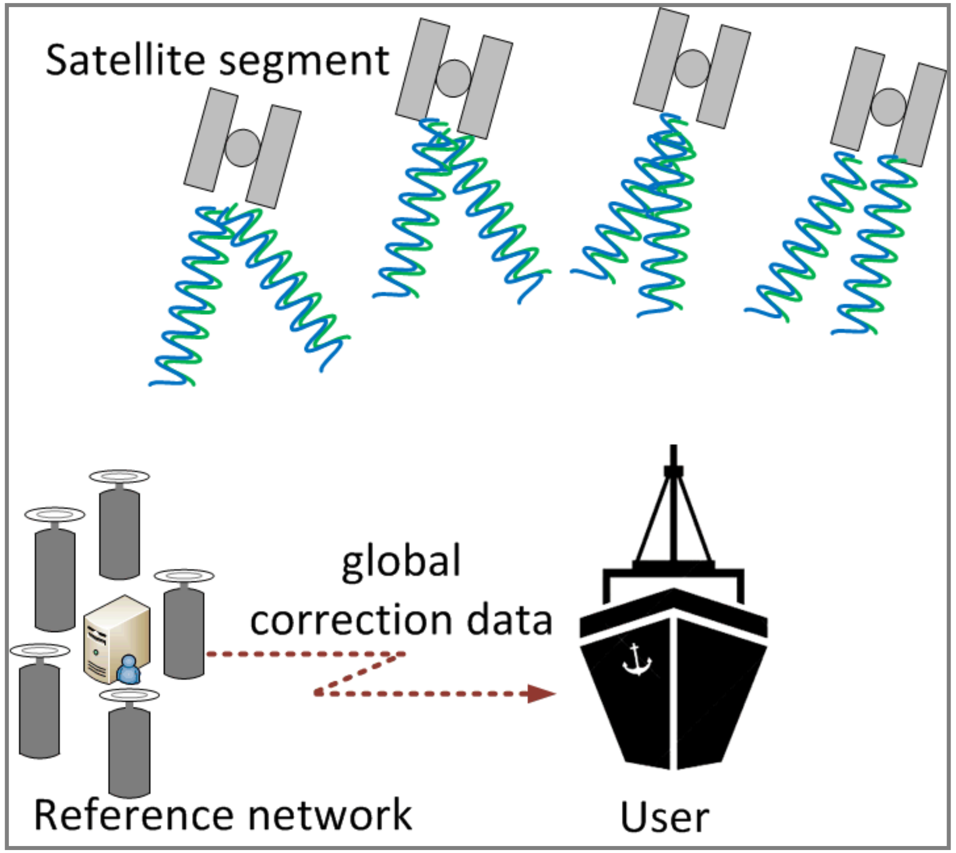




Medina, D., Vilà-Valls, J., Chaumette, E., Vincent, F., & Closas, P. (2021). Cramér-Rao bound for a mixture of real-and integer-valued parameter vectors and its application to the linear regression model. *Signal Processing*, *179*, 107792.

Precise Point Positioning

Precise Point Positioning (PPP)



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On PPP corrections

Precise ephemeris

Type		Accuracy	Latency	Sample interval
Broadcast	Orbits	~ 100 cm	Real time	daily
	Sat clocks	~ 5 ns		
Ultra-Rapid	Orbits	~3cm	3-9 hours	15 min
	Sat clocks	~ 150 ps		
Rapid	Orbits	~ 2.5 cm	17-41 hours	15 min
	Sat clocks	~ 75 ps		5 min
Final	Orbits	~ 2.5cm	12-18 days	15 min
	Sat clocks	~ 75 ps		30 s

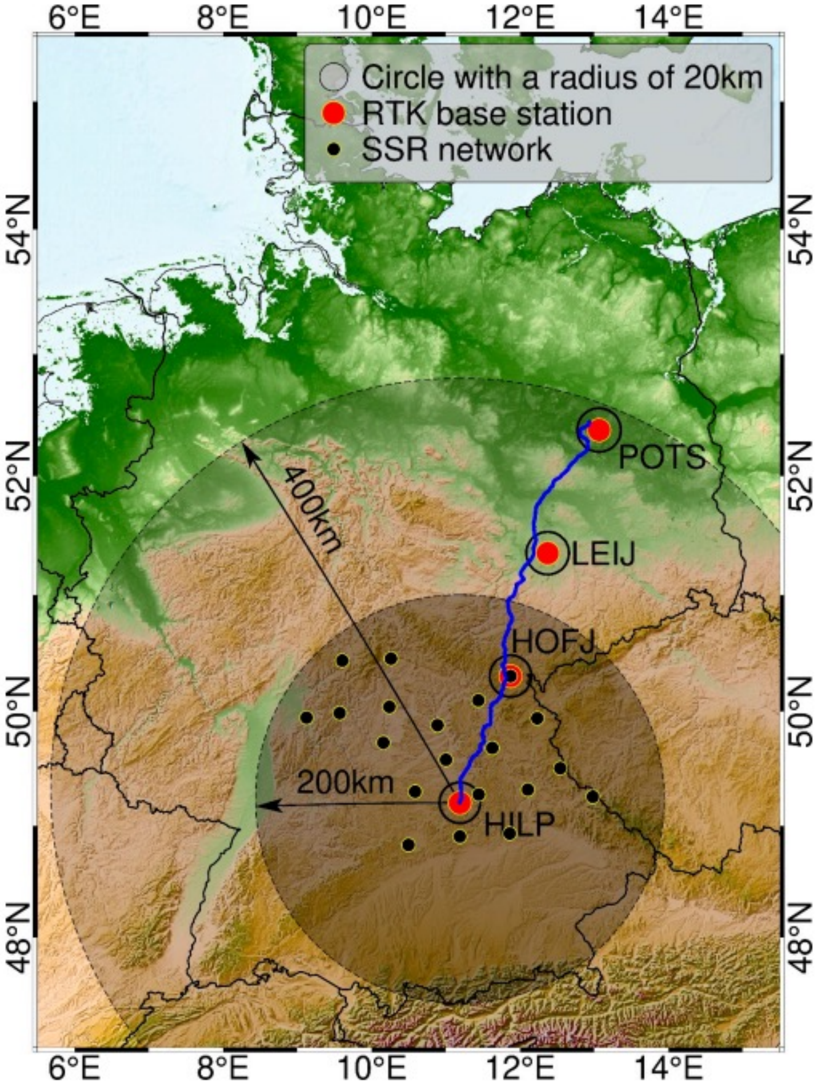
International GNSS Service, Products. Link: http://www.igs.org/products

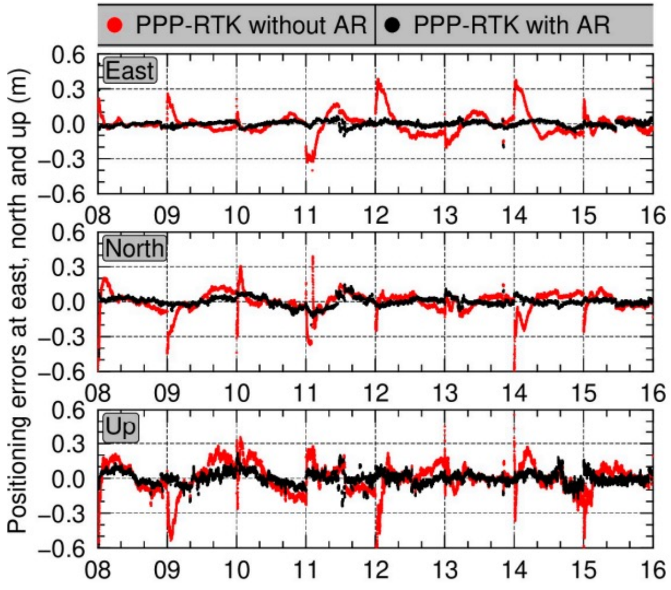
The new trend: PPP-RTK / PPP-Ambiguity Resolution

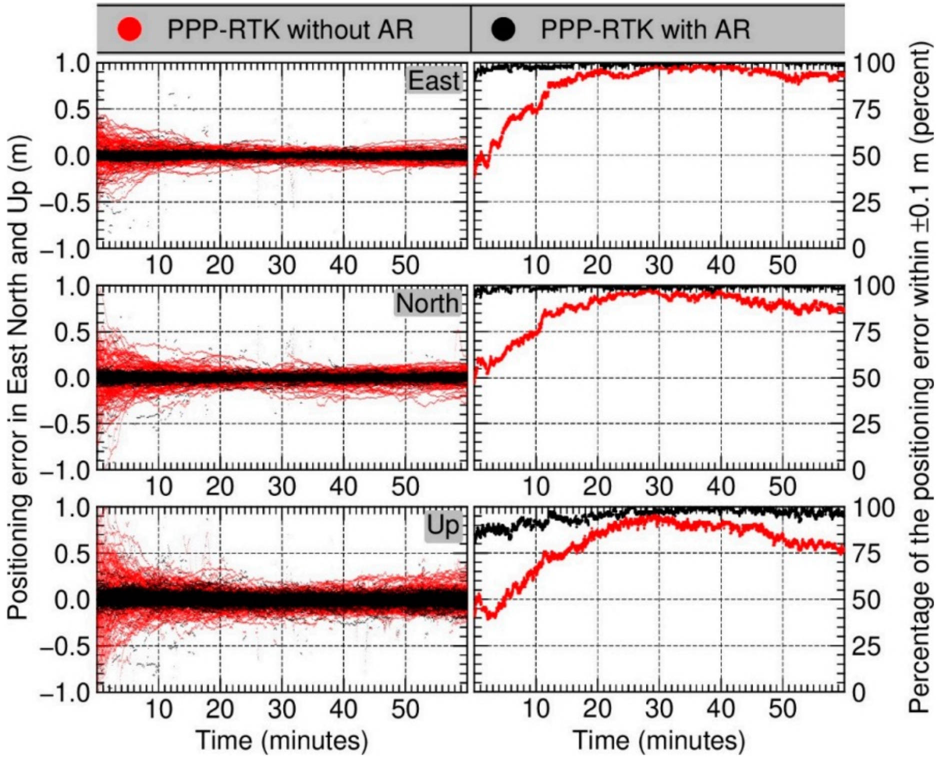
- Hybrid between PPP & RTK with the aim to achieve high precision solutions *fast* and *worldwide:*
  - The core is a PPP engine fed with precise ephemeris and regional/local atmospheric information
  - Single differencing (wrt. a pivot satellite) is applied to eliminate the receiver’s carrier biases
- Galileo High Accuracy Service (HAS) is a great effort from the EU to make

Growing interest to make high precision GNSS available for the mass market → addressing carrier phase issues is key!

PPP-RTK in action!



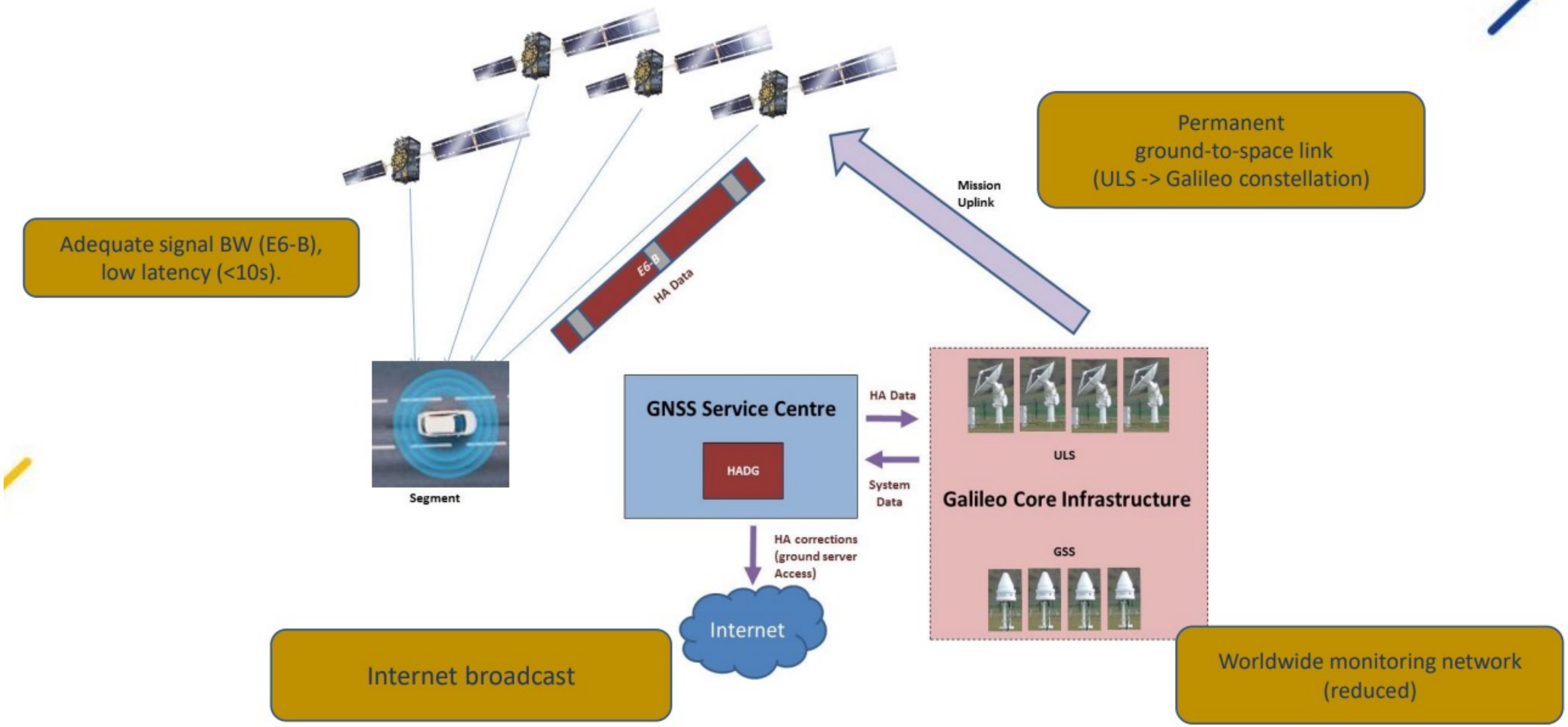




An, X., Ziebold, R., & Lass, C. (2023). PPP-RTK with rapid convergence based on SSR corrections and its application in transportation. Remote Sensing, 15(19), 4770.

An, X., Ziebold, R., & Lass, C. (2023). From RTK to PPP-RTK: towards real-time kinematic precise point positioning to support autonomous driving of inland waterway vessels. GPS Solutions, 27(2), 86.

Briefly on Galileo High Accuracy Service (HAS)



Adequate signal BW (E6-B), low latency (<10s).

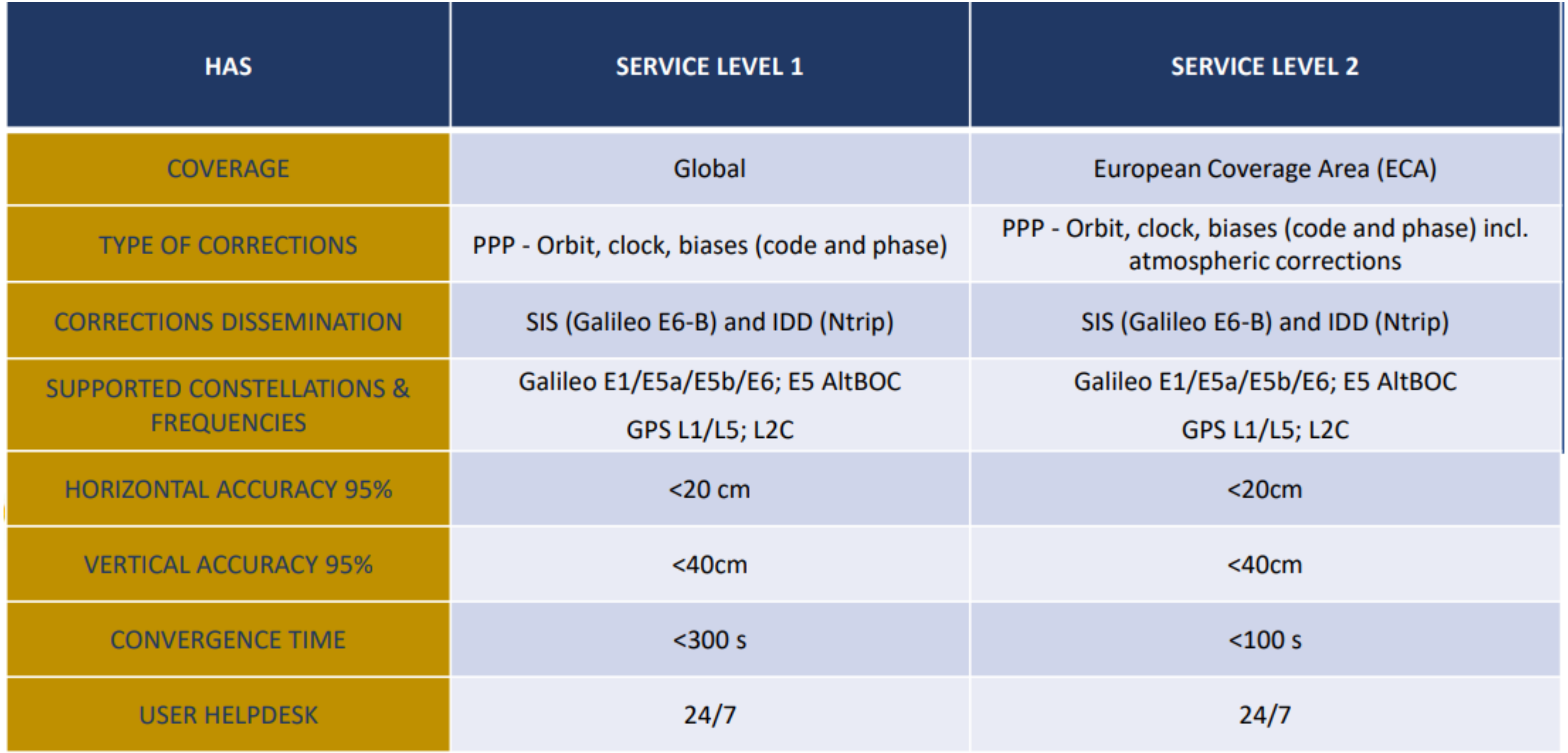
Permanent ground-to-space link (ULS -> Galileo constellation)

Internet broadcast

Worldwide monitoring network (reduced)

Source: EUSPA

Briefly on Galileo HAS



HAS	SERVICE LEVEL 1	SERVICE LEVEL 2
COVERAGE	Global	European Coverage Area (ECA)
TYPE OF CORRECTIONS	PPP - Orbit, clock, biases (code and phase)	PPP - Orbit, clock, biases (code and phase) incl. atmospheric corrections
CORRECTIONS DISSEMINATION	SIS (Galileo E6-B) and IDD (Ntrip)	SIS (Galileo E6-B) and IDD (Ntrip)
SUPPORTED CONSTELLATIONS & FREQUENCIES	Galileo E1/E5a/E5b/E6; E5 AltBOC GPS L1/L5; L2C	Galileo E1/E5a/E5b/E6; E5 AltBOC GPS L1/L5; L2C
HORIZONTAL ACCURACY 95%	<20 cm	<20cm
VERTICAL ACCURACY 95%	<40cm	<40cm
CONVERGENCE TIME	<300 s	<100 s
USER HELPDESK	24/7	24/7

Source: EUSPA

Recap on high precision GNSS

- As of today, **RTK is the most used technique for high precision**
  - Instantaneous cm (or even mm) level accuracy
  - Requires nearby base stations + low latency and “broad” communication channel
- **PPP allows for global positioning with dm-accuracy**
  - A “long” convergence time is required to achieve high precision
  - Real time applicability is limited by the correction services
- **PPP-RTK is likely to be the future** & we are in the time and place to make that a reality!
- Estimation bounds for PPP / PPP-RTK are not yet derived…

Outline

1 What are Carrier Phase Observations?
  - Carrier Phase Limitations

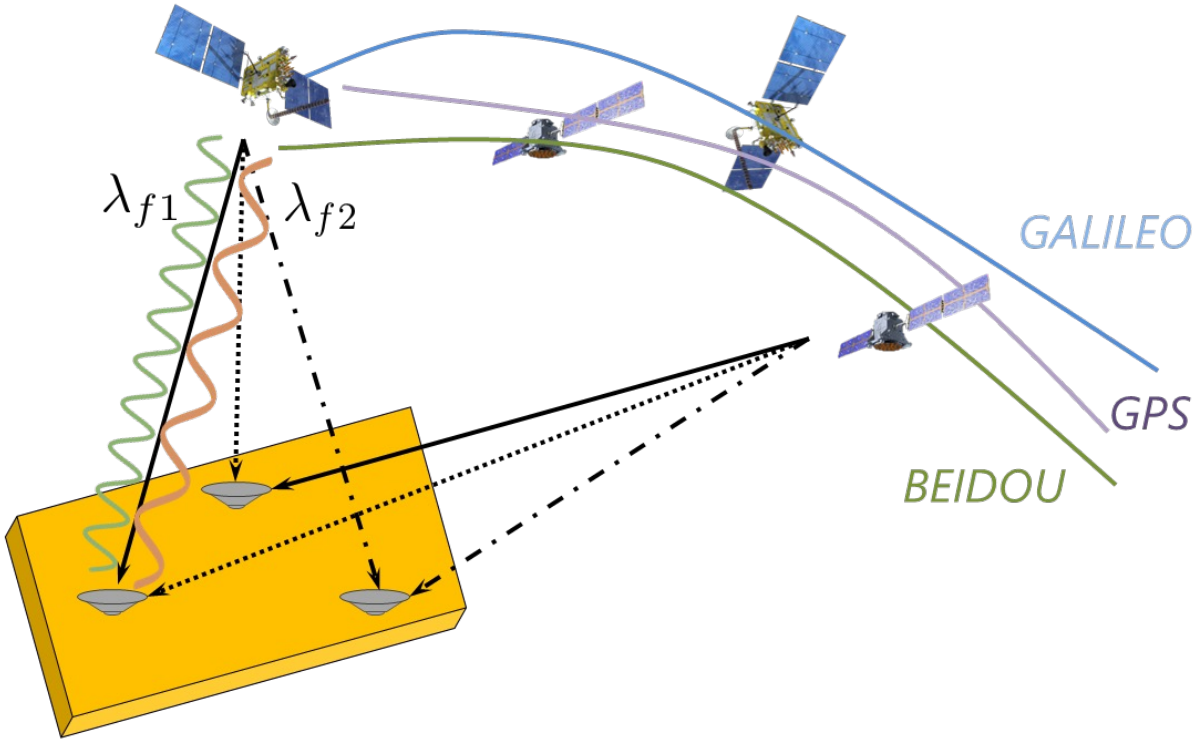
2 Precise Positioning Techniques
  - Real Time Kinematics (RTK)
  - Precise Point Positioning (PPP)

3 Multi-Antenna Systems

4 Cooperative GNSS Positioning

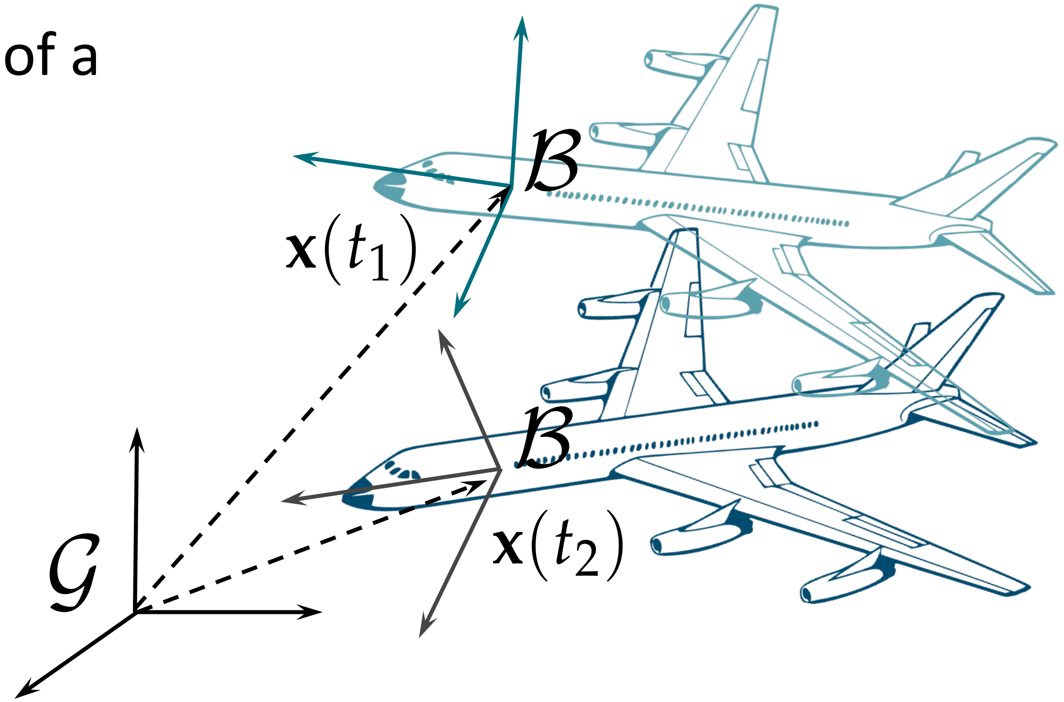
Multi-Antenna Applications

Estimating a vehicle's pose



Precise attitude estimation
Dealing with multiple antennas and integer ambiguities

- **Attitude Determination** → the orientation of a vehicle wrt. a reference frame
- Using multi-antenna setups → „absolute“ „drift-less“ attitude information
- **Orientation precision** depends on:
  - Inter-antenna separation
  - Differential positioning error



- Carrier phase observations & Integer Ambiguity Resolution (**IAR**) is key
- The abundance of measurements may complicate things…

High Precision GNSS Techniques

	Single antenna	Multi antenna
Recursive	**Precise Point Positioning (PPP)**	**Array PPP**
Snapshot	**Real Time Kinematic (RTK)**	**Joint Position and Attitude (JPA)** **GNSS Attitude**

GNSS for attitude estimation

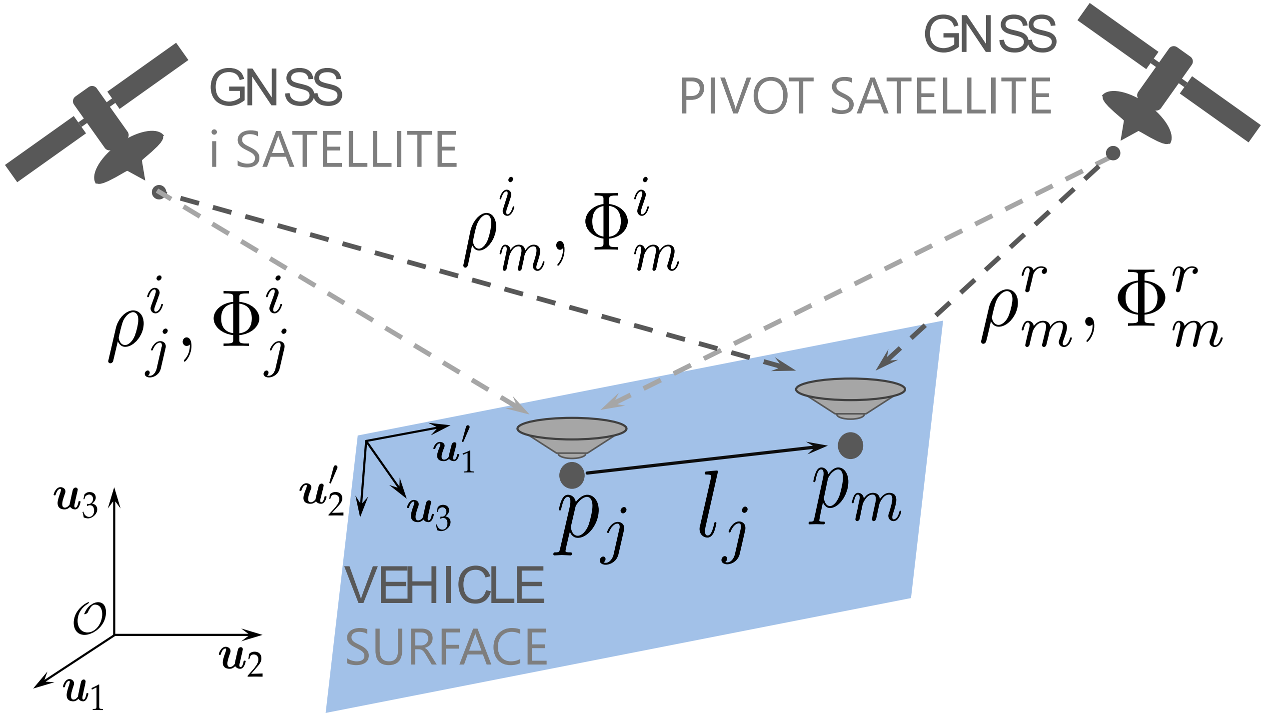
GNSS-based Attitude Determination requires:

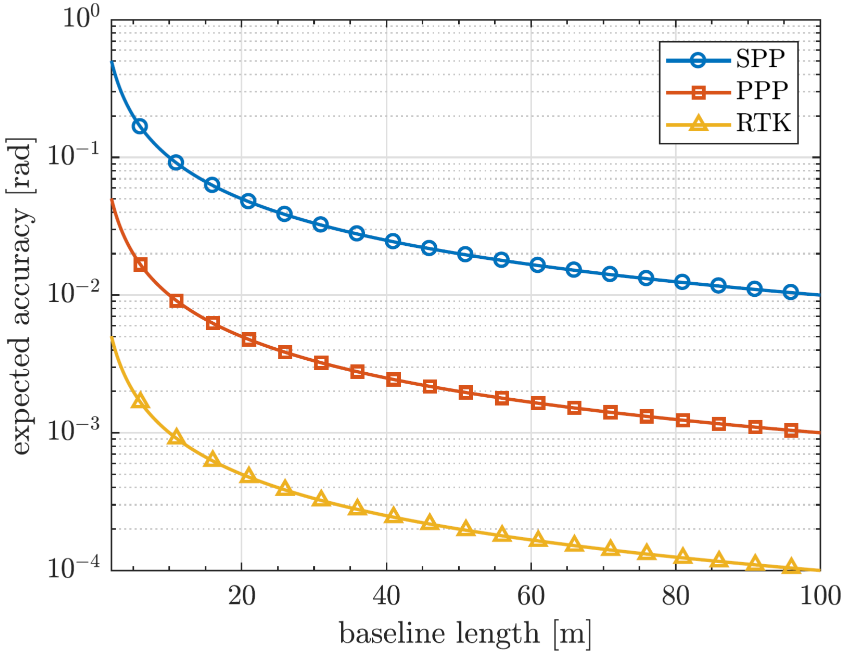
- Multi – antenna setup
- Surveyed antennas’ position in the local frame

Attitude accuracy depends on:

- Antenna separation
- Positioning accuracy

$$\sigma_{\phi} = \frac{\sigma_{\delta p}}{\|l\|^2}$$





GNSS Attitude Model

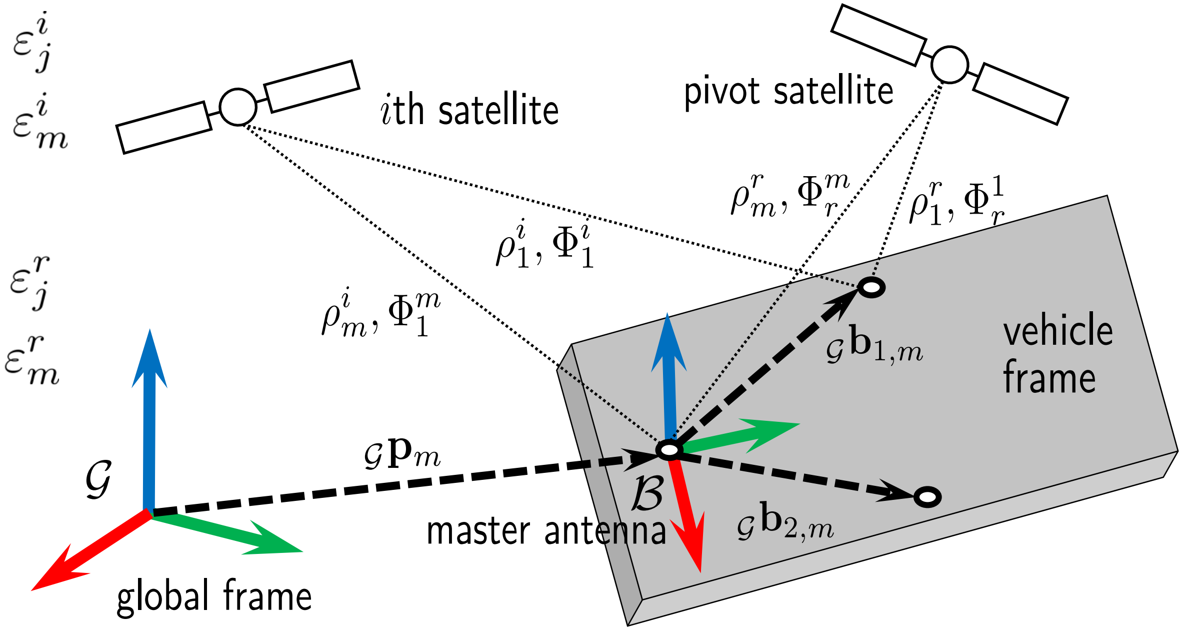
m – master antenna; *r* – pivot satellite
N+1 – antennas; *n+1* – satellites
M – total number of code/phase obs.

$$\Phi_j^i = \|p^i - p_j\| - I^i + T^i + c\left(-dt^i + dt_j\right) + \lambda N_j^i + \varepsilon_j^i$$

$$(-)\quad \Phi_m^i = \|p^i - p_m\| - I^i + T^i + c\left(-dt^i + dt_m\right) + \lambda N_m^i + \varepsilon_m^i$$

$$\Phi_j^r = \|p^r - p_j\| - I^r + T^r + c\left(-dt^r + dt_j\right) + \lambda N_j^r + \varepsilon_j^r$$

$$(-)\quad \Phi_m^r = \|p^r - p_m\| - I^r + T^r + c\left(-dt^r + dt_m\right) + \lambda N_m^r + \varepsilon_m^r$$



Set of observations

$$\mathbf{y} = \left[\boldsymbol{DD\Phi}_1^\top, \ldots, \boldsymbol{DD\Phi}_N^\top, \boldsymbol{DD\rho}_1^\top, \ldots, \boldsymbol{DD\rho}_N^\top\right]$$

The *Mixed Attitude Model*

$$\mathbf{y} \sim \mathcal{N}\left(\mathbf{A}\mathbf{a} + \mathbf{h}(\mathbf{q}), \boldsymbol{\Sigma}\right),\ \mathbf{a} \in \mathbb{Z}^M, \mathbf{q} \in \mathcal{S}^3$$

The GNSS-Based Attitude Model

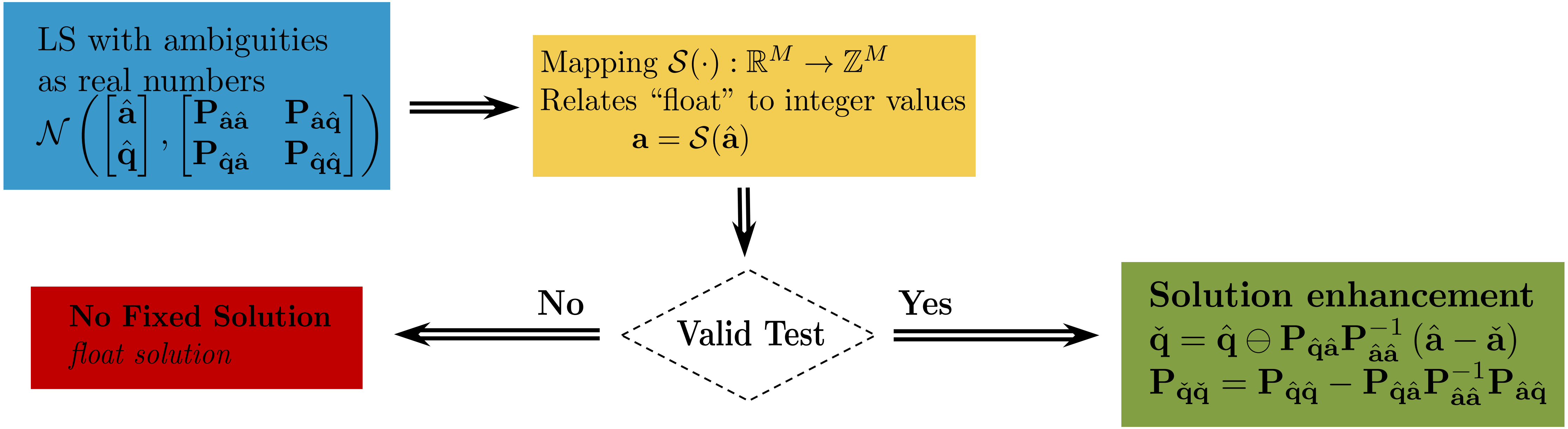
Solving the puzzle

$$\|\hat{\mathbf{e}}\|^2_{\boldsymbol{\Sigma}} + \min_{\mathbf{a}\in\mathbb{Z}^M} \left(\|\hat{\mathbf{a}} - \mathbf{a}\|^2_{\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}} + \min_{\mathbf{q}\in\mathcal{S}^3} \|\hat{\mathbf{q}}(\mathbf{a}) - \mathbf{q}\|^2_{\mathbf{P}_{\hat{\mathbf{q}}(\mathbf{a})}} \right)$$

Float solution

IAR

Fixed solution



LS with ambiguities as real numbers

$$\mathcal{N}\left(\begin{bmatrix}\hat{\mathbf{a}}\\ \hat{\mathbf{q}}\end{bmatrix}, \begin{bmatrix}\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} & \mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{q}}}\\ \mathbf{P}_{\hat{\mathbf{q}}\hat{\mathbf{a}}} & \mathbf{P}_{\hat{\mathbf{q}}\hat{\mathbf{q}}}\end{bmatrix}\right)$$

Mapping $\mathcal{S}(\cdot) : \mathbb{R}^M \rightarrow \mathbb{Z}^M$

Relates “float” to integer values

$$\mathbf{a} = \mathcal{S}(\hat{\mathbf{a}})$$

Valid Test

No

No Fixed Solution

float solution

Yes

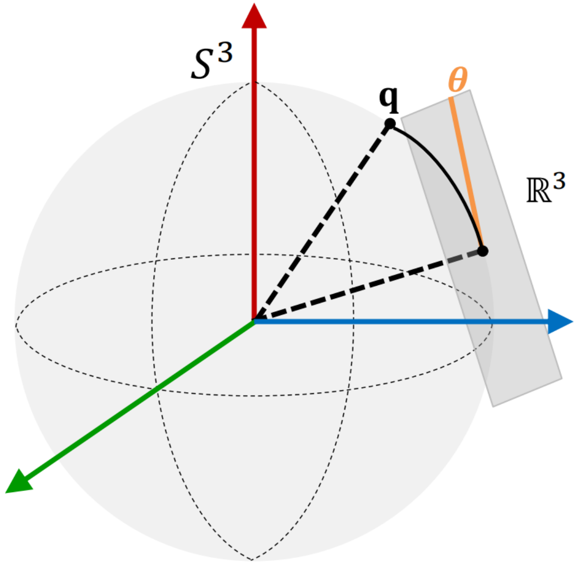
Solution enhancement

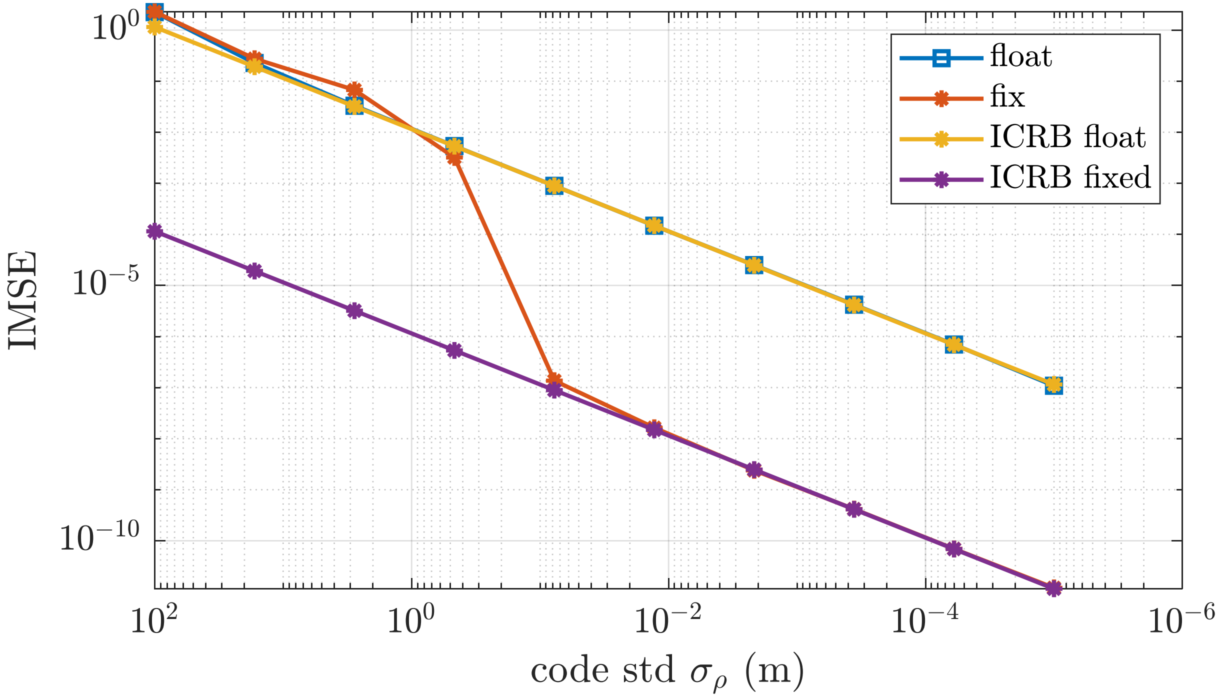
$$\check{\mathbf{q}} = \hat{\mathbf{q}} \ominus \mathbf{P}_{\hat{\mathbf{q}}\hat{\mathbf{a}}}\mathbf{P}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}\left(\hat{\mathbf{a}} - \check{\mathbf{a}}\right)$$

$$\mathbf{P}_{\check{\mathbf{q}}\check{\mathbf{q}}} = \mathbf{P}_{\hat{\mathbf{q}}\hat{\mathbf{q}}} - \mathbf{P}_{\hat{\mathbf{q}}\hat{\mathbf{a}}}\mathbf{P}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}\mathbf{P}_{\hat{\mathbf{a}}\hat{\mathbf{q}}}$$

Estimation Bounds for the GNSS Attitude Model

- Even more challenging than RTK (mixed real and integer)
- The GNSS Attitude Model involves: **Lie Group SO(3)** + **Integers**
- Luckily, our dear Samy Labsir is a CRB-derivation machine → Intrinsic CRB for the attitude model





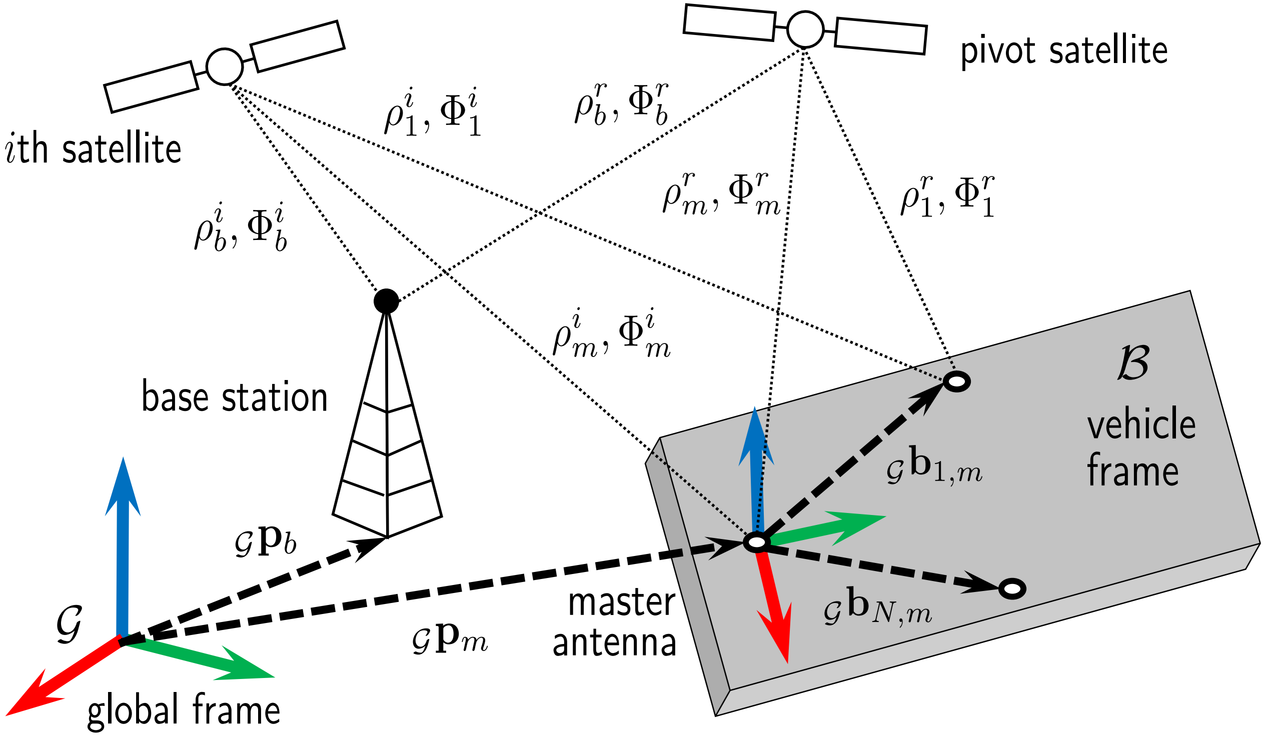
Labsir, S., Medina, D., Vilà-Valls, Renaux, A., J., Chaumette (to be submitted). CRB for Lie Group x Integers and Its Application to Carrier Phase-Based GNSS Attitude Estimation.

Joint Position and Attitude (JPA) Model

- JPA estimation → solving the navigation for a vehicle with multiple antennas and access to a base station

The goal

1. Exploit the knowledge on the antennas' configuration & noise cross-correlation
2. Propose snapshot and recursive estimators for the JPA problem
3. Increase availability of precise orientation and positioning

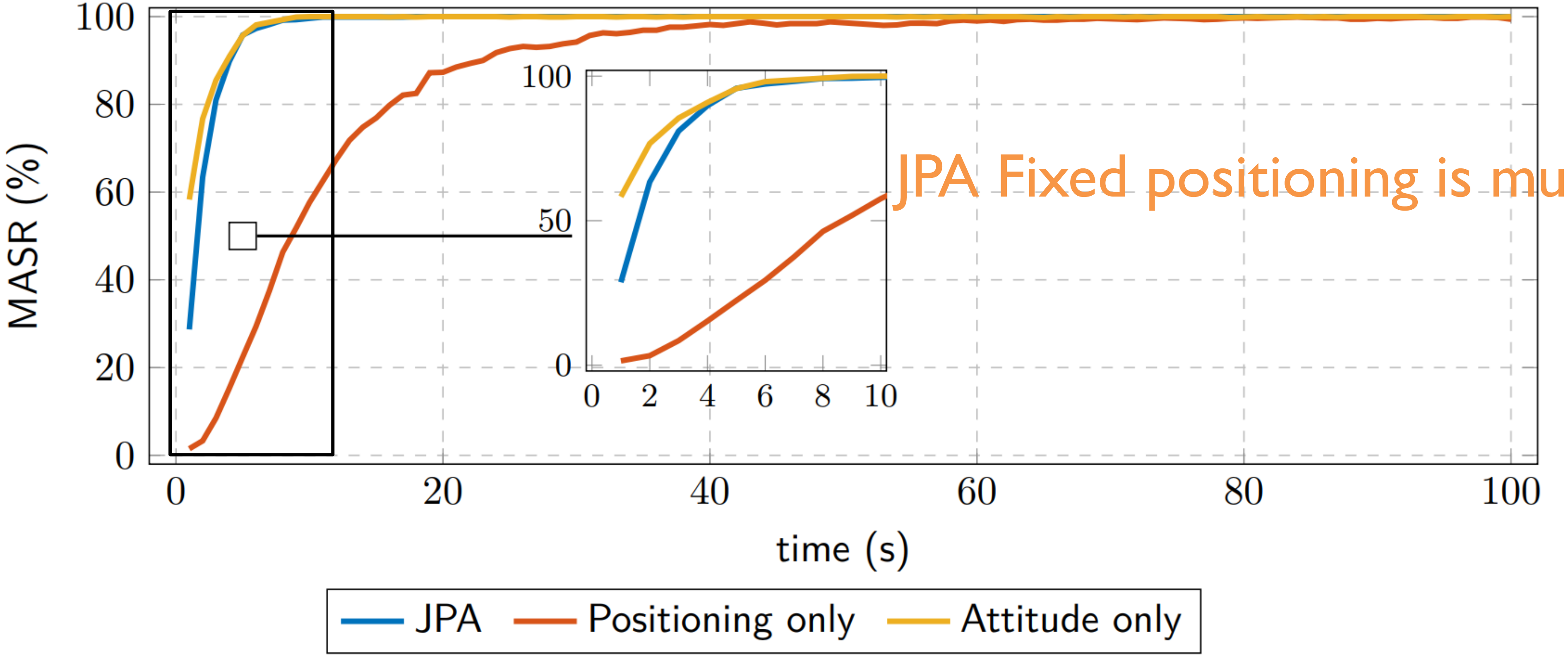


Medina, D., Vilà-Valls, J., Hesselbarth, A., Ziebold, R., & García, J. (2020). On the recursive joint position and attitude determination in multi-antenna GNSS platforms. Remote Sensing, 12(12), 1955.

JPA in action

- **Monte Carlo-based simulation to address JPA against RTK & GNSS Attitude**
- A multi-antenna setup with 4 antennas separated by 1 meter
- The initial distance to the base station is 5 km

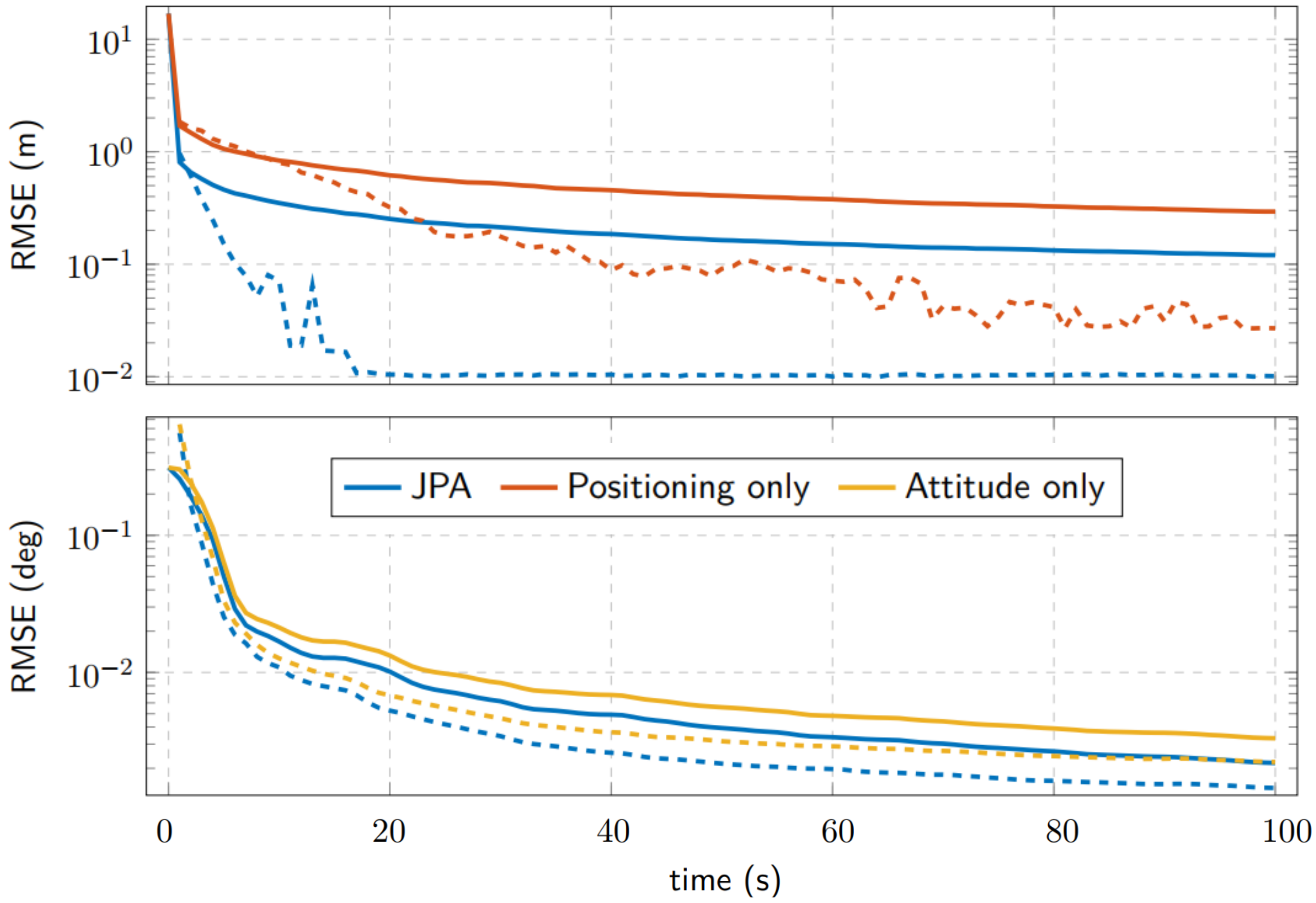
Mean ambiguity success rate (MASR) **[Up and left is better]**



JPA Fixed positioning is much faster!

JPA in action

[Down and left is better]



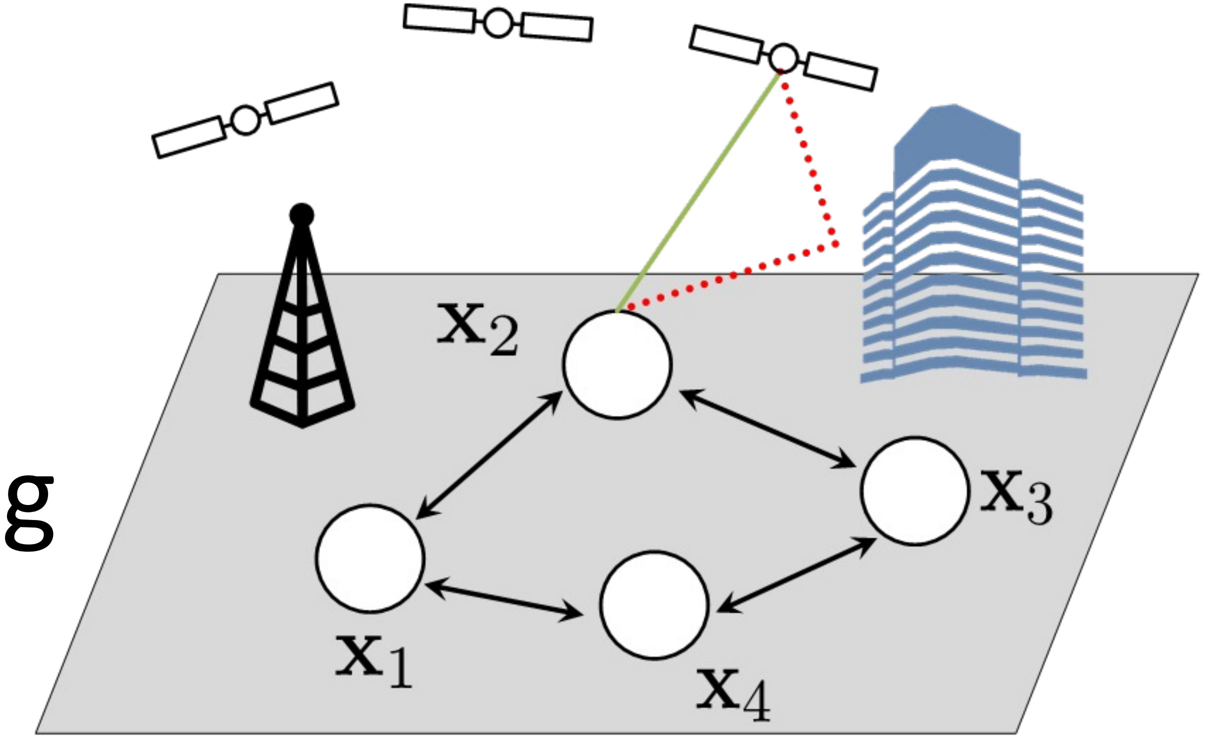
In summary…

- JPA leads to higher accuracy → both for positioning and attitude!
- Positioning greatly improves in availability and time-to-fix

Outline

Cooperative Positioning

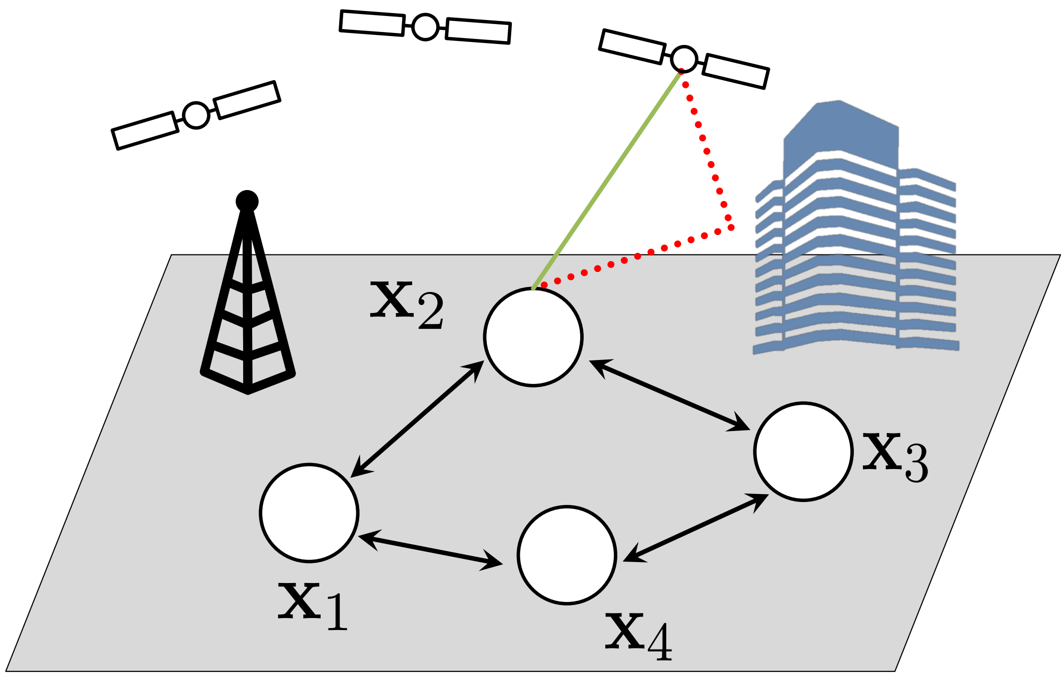
Network of users helping each other



Collaborative Positioning

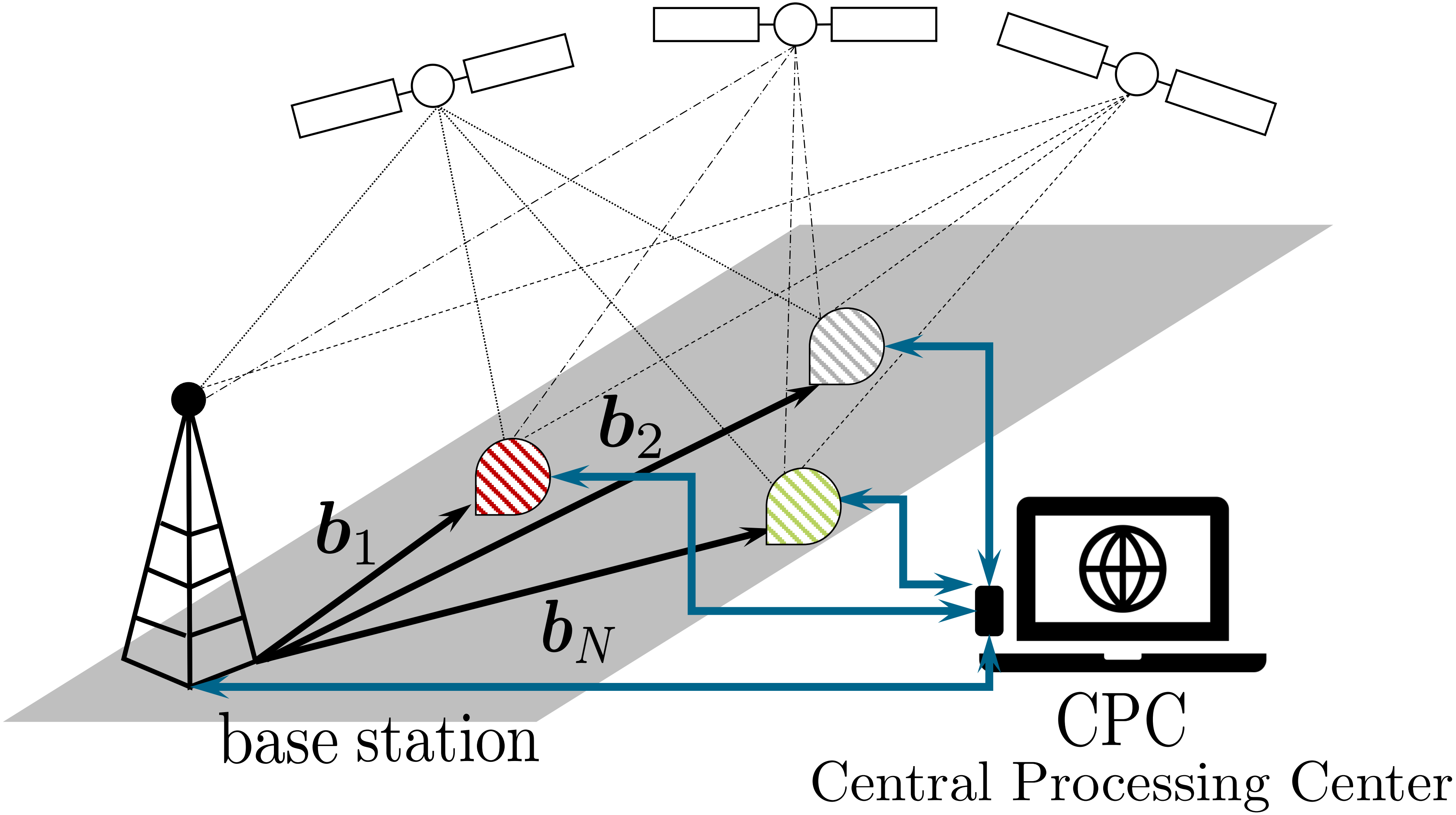
Overcoming the limitations for RTK

- **Real Time Kinematic (RTK)** is the standard for high precision positioning
- RTK underperforms in urban scenarios: limited visibility, multipath effects, distance to stations
- **Collaborative approaches** → paradigm for connected vehicles, helpful for GNSS limitations
- Collaboration understood from different prisms
  - Active collaboration: inter-agent ranging & localization exchange
  - Passive collaboration: broadcast of observations

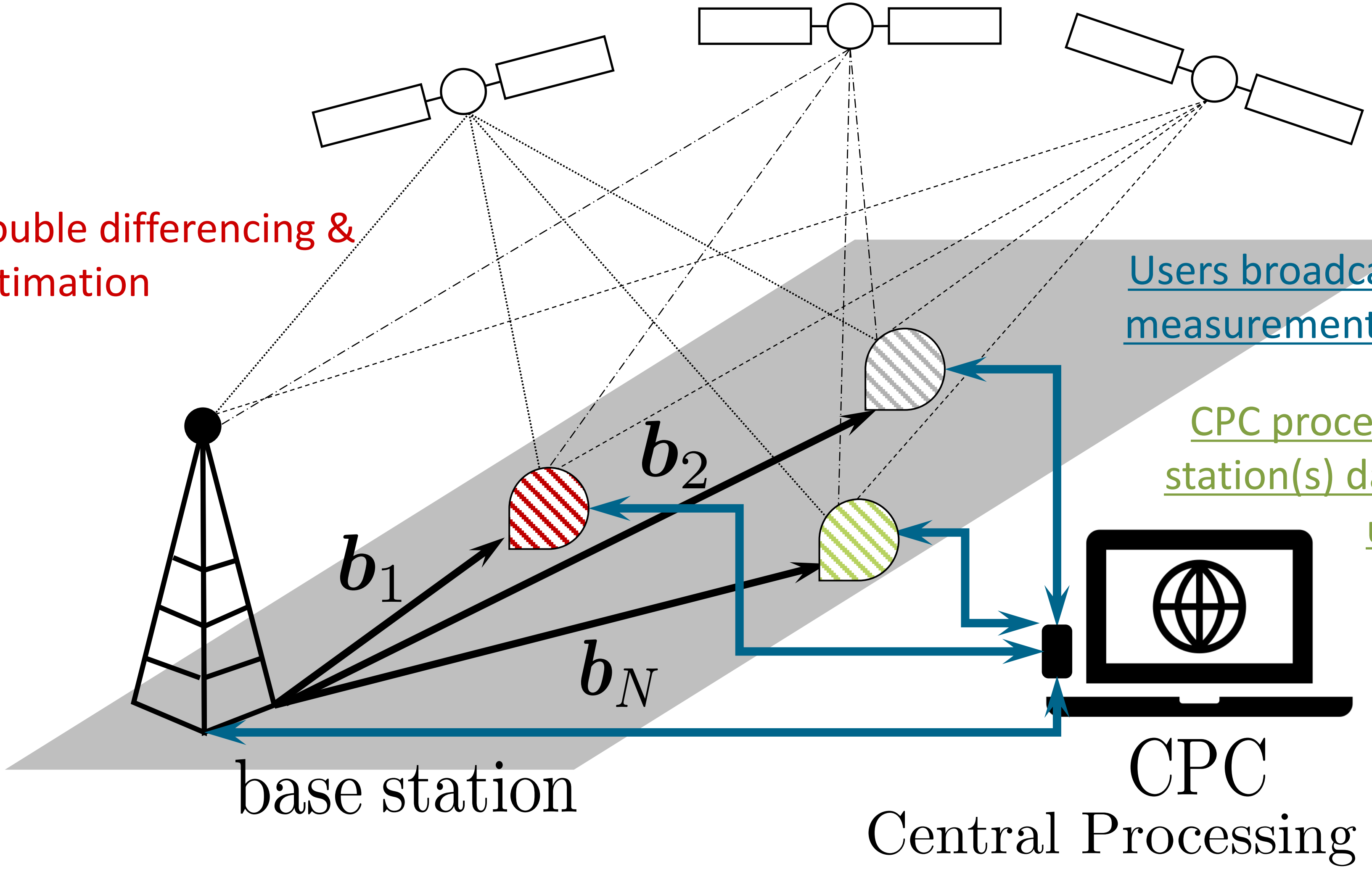


- **Collaborative RTK (C-RTK) → concept for high precision positioning with passive collaboration**

Collaborative-RTK (C-RTK) – Conceptual & Technical Idea



Collaborative-RTK (C-RTK) – Conceptual & Technical Idea



Processing → double differencing & mixed model estimation

Users broadcast/send the measurements to the CPC

CPC processes users and base station(s) data → batch estimate unknowns

C-RTK – Positioning Problem

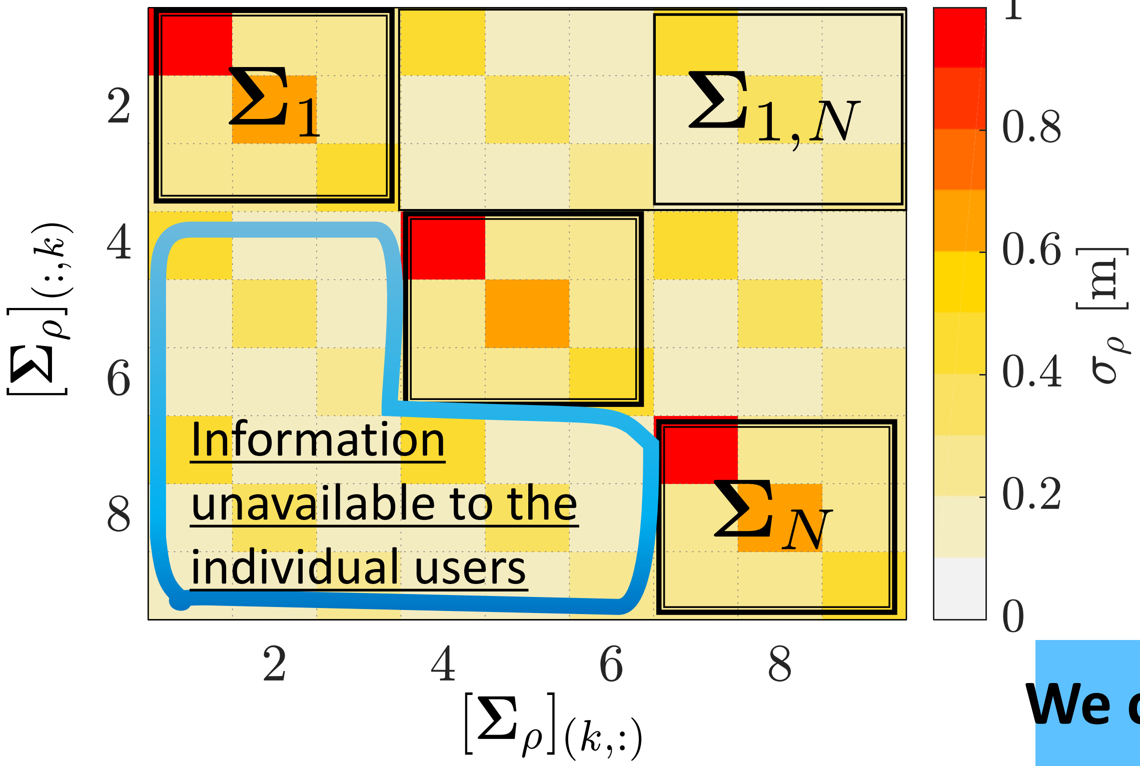
Conventional RTK

$$\boldsymbol{y}_j \sim \mathcal{N}\left(\boldsymbol{A}\boldsymbol{a}_j + \boldsymbol{B}\boldsymbol{b}_j, \boldsymbol{\Sigma}_j\right)$$

for *N* users →

Collaborative RTK

$$\tilde{\boldsymbol{y}} \sim \mathcal{N}\left(\tilde{\boldsymbol{A}}\tilde{\boldsymbol{a}} + \tilde{\boldsymbol{B}}\tilde{\boldsymbol{b}}, \tilde{\boldsymbol{\Sigma}}\right), \ \tilde{\boldsymbol{a}} \in \mathbb{Z}^{n \cdot N}, \tilde{\boldsymbol{b}} \in \mathbb{R}^{3 \cdot N}$$



Information unavailable to the individual users

"Extended" version of obs., unknowns, matrices

$$\tilde{\boldsymbol{y}} = \left[\boldsymbol{DD\Phi}_1^\top, \ldots, \boldsymbol{DD\Phi}_N^\top, \boldsymbol{DD\rho}_1^\top, \ldots, \boldsymbol{DD\rho}_N^\top\right]^\top$$

$$\tilde{\boldsymbol{a}} = \left[\boldsymbol{a}_1^\top, \ldots, \boldsymbol{a}_N^\top\right]^\top, \ \tilde{\boldsymbol{b}} = \left[\boldsymbol{b}_1^\top, \ldots, \boldsymbol{b}_N^\top\right]^\top$$

The importance of stochastic modeling

- The cross-correlations due to combining observations wrt. base station → fundamental information!

We can leverage on the existing CRBs and estimators for the mixed model problem

C-RTK – Overview, Benefits, Limitations

- Regular RTK: involves base station to users communication
- C-RTK is a centralized, passive collaboration architecture

✓ **Privacy preserving:** users do not compromise their localization information

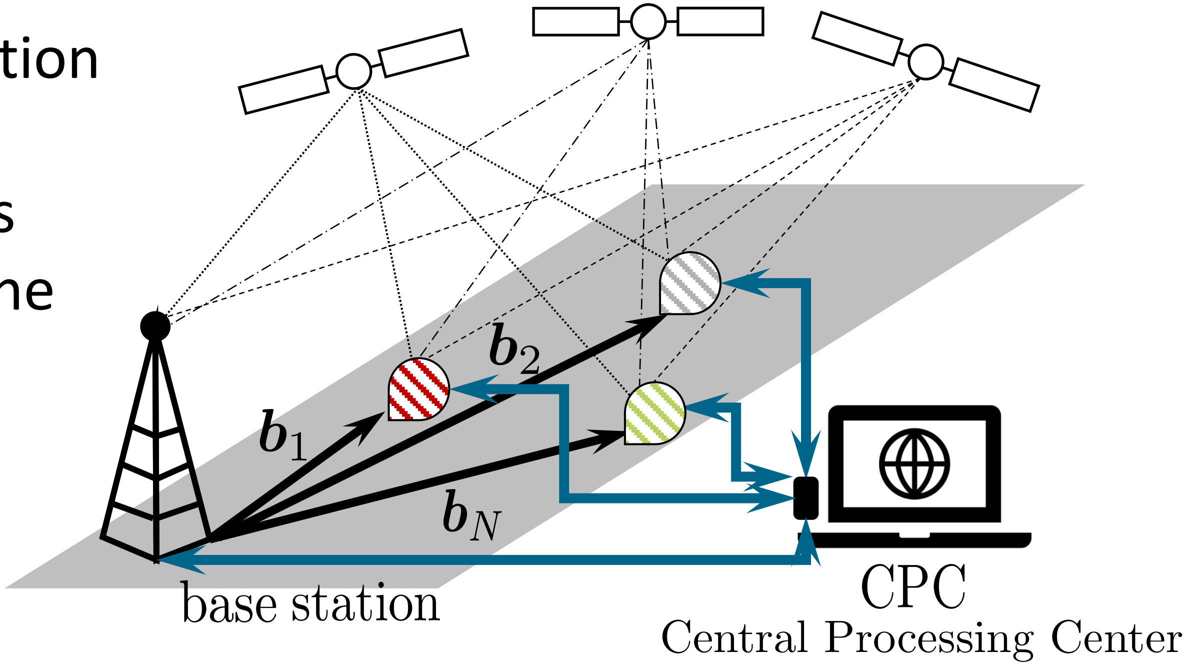
✓ Estimation process benefit from all available information

✗ A low-latency, "broad" 2-way communication channel is needed

✗ Dealing with asynchronous measurements

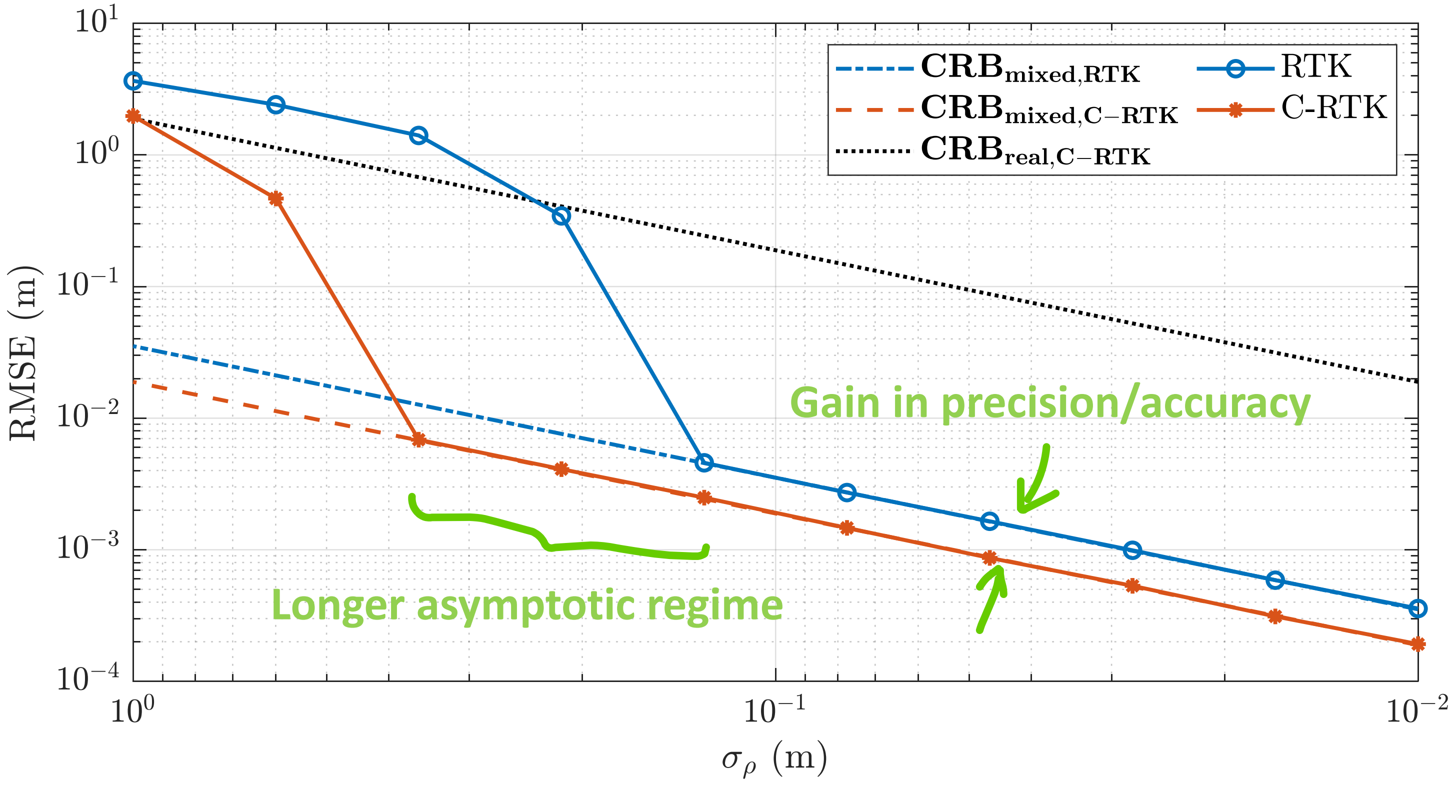
✗ Growing computational complexity with the number of users

? What is the performance gain?



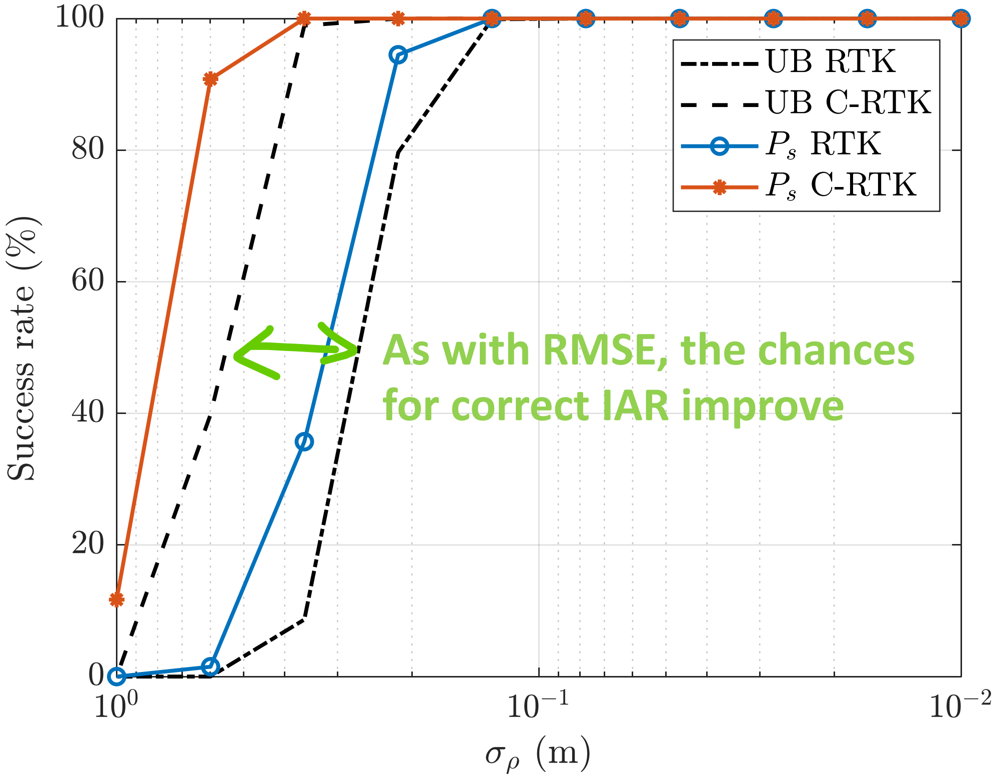
Monte Carlo based Performance Analysis

The information gain in C-RTK → superior performance wrt. RTK



Gain in precision/accuracy

Longer asymptotic regime



As with RMSE, the chances for correct IAR improve

Medina, D., Calatrava, H., Castro-Arvizu, J. M., Closas, P., & Vila-Valls, J. (2023, April). A Collaborative RTK Approach to Precise Positioning for Vehicle Swarms in Urban Scenarios. In 2023 IEEE/ION Position, Location and Navigation Symposium (PLANS) (pp. 254-259).

1 What are Carrier Phase Observations?

- Carrier Phase Limitations

2 Precise Positioning Techniques

- Real Time Kinematics (RTK)
- Precise Point Positioning (PPP)

3 Multi-Antenna Systems

4 Industry and research perspectives

Outline

Industry and Research Perspectives

From industry:

- LEO navigation is becoming a thing & carrier phase is also involved
- Certification (integrity monitoring) is the last step before GNSS is truly everywhere
- Other interesting applications: collaborative or lunar positioning

From research:

- There is a need for better (robust!) estimators
- Multi sensor integration (with cameras, LiDARs, etc.) is still challenging
- What is the role of Machine Learning? How to successfully deploy it for GNSS?
- Further on architectures and solutions for Coop. GNSS → ITSNT 27th June

Seminar on 5th July

- GNSS is a multi-billion industry with unlimited perspectives
- Knowledge on high precision GNSS techniques is one of the most wanted skills!

Say hi at:
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Thanks for your attention!

Impressum

Title: **High Precision Satellite-based Navigation**
Theory and Applications

Date: 2024-06-14

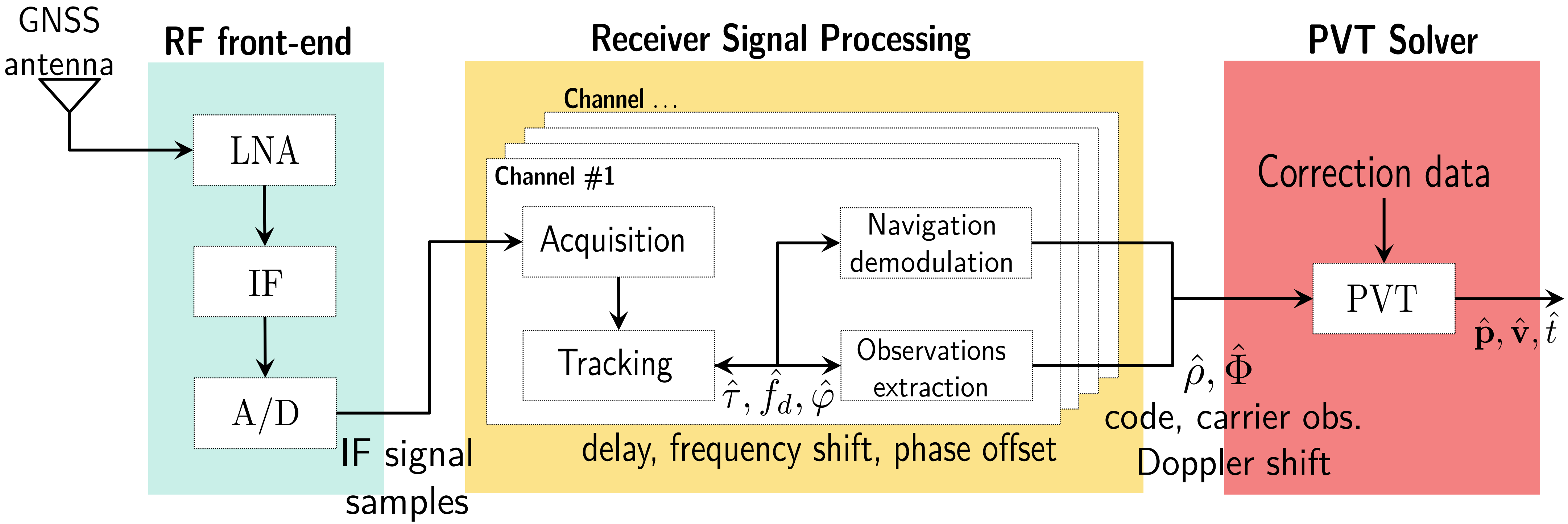
Author: Daniel Medina (daniel.ariasmedina@dlr.de)

Institut: Communications and Navigation

Credits: All pictures are „DLR (CC BY-NC-ND 3.0)“,
unless otherwise stated

BACK UP SLIDES

General GNSS Receiver Architecture



RTK Processing

State estimate

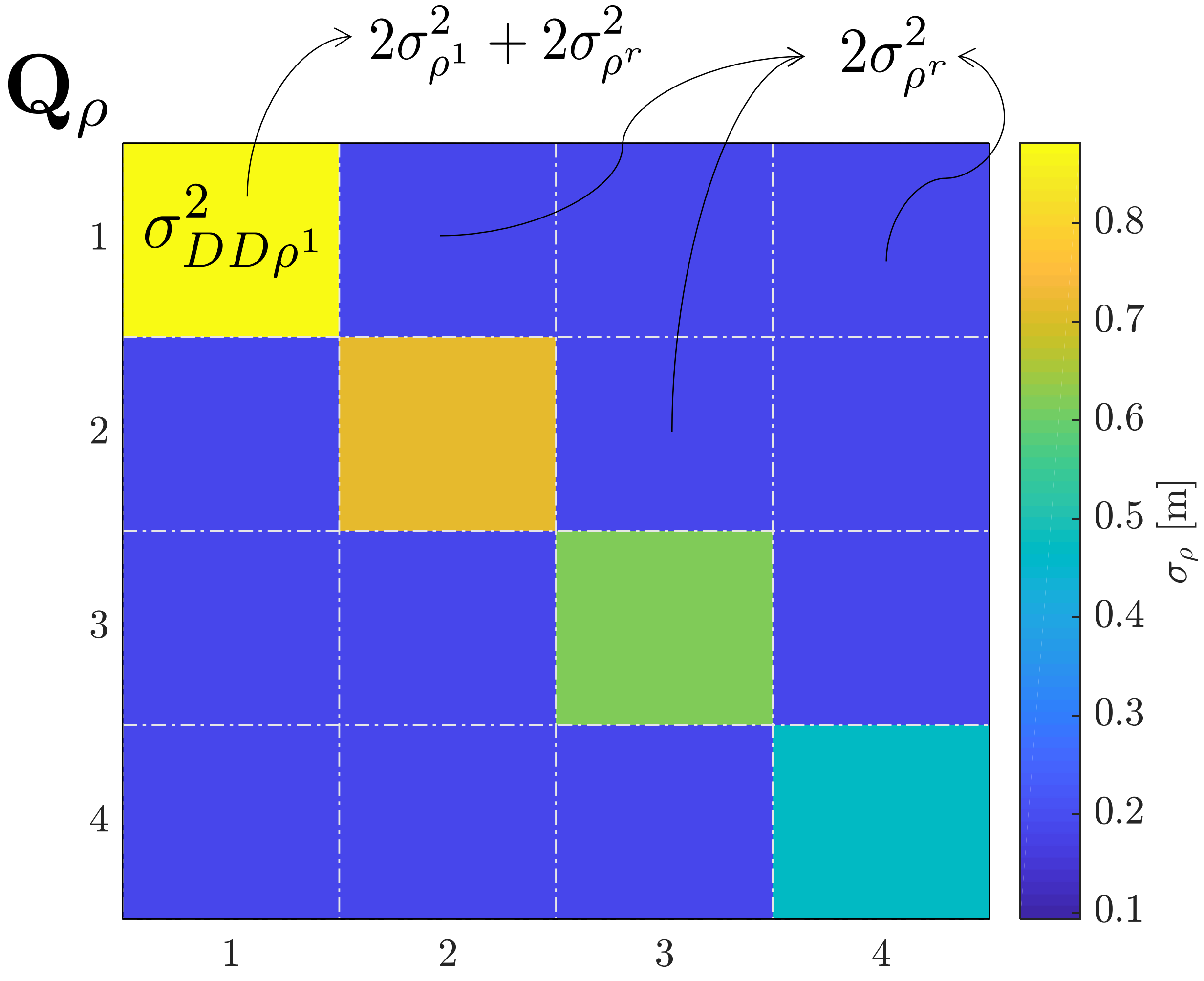
$$\mathbf{x} = \begin{bmatrix}\mathbf{a} \\ \mathbf{b}\end{bmatrix}, \quad \mathbf{a} \in \mathbb{Z}^n, \ \mathbf{b} \in \mathbb{R}^3$$

Set of observations

$$\mathbf{y} = \begin{bmatrix}\boldsymbol{DD\Phi} \\ \boldsymbol{DD\rho}\end{bmatrix}, \quad \mathbf{y} \in \mathbb{R}^{2n}$$

Careful with noise statistics

$$\boldsymbol{\eta} \sim \mathcal{N}\left(\mathbf{0}_{2n,1}, \underbrace{\begin{bmatrix}\mathbf{Q}_\Phi & \\ & \mathbf{Q}_\rho\end{bmatrix}}_{\mathbf{Q}_y}\right)$$



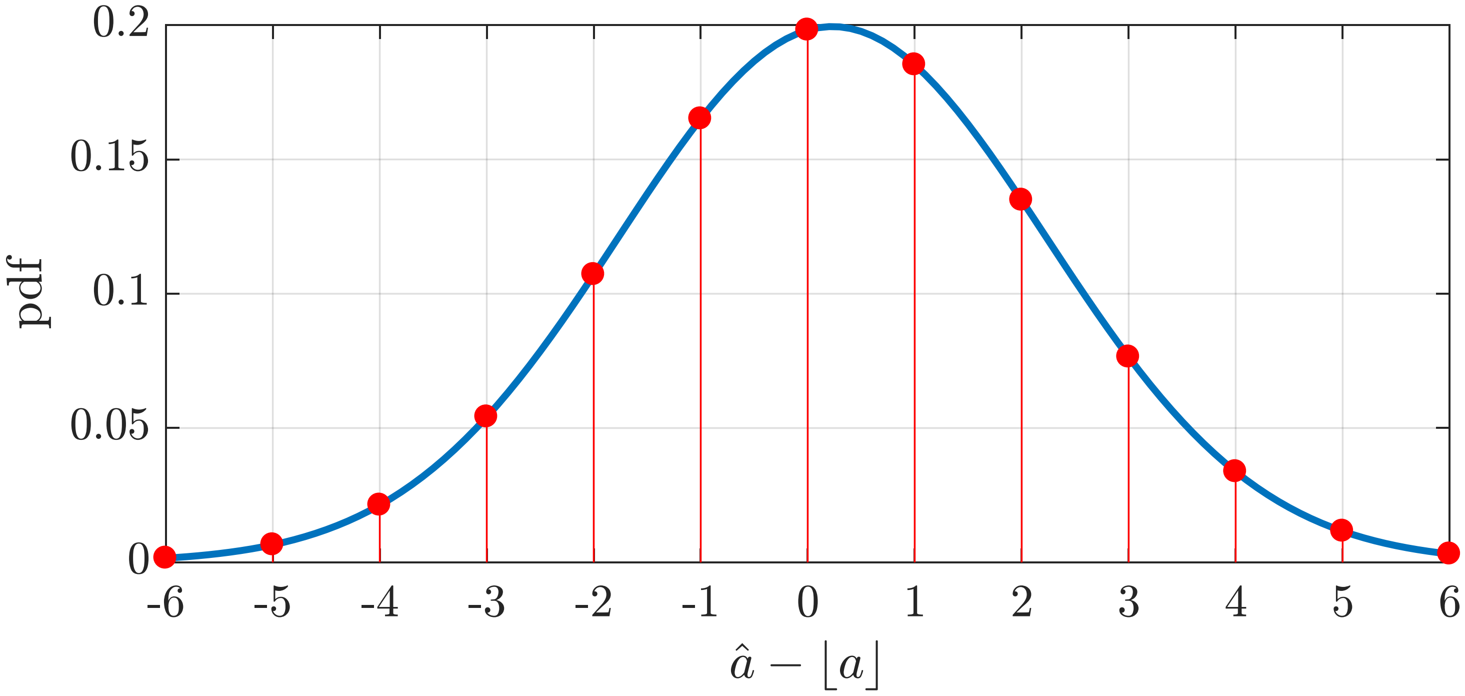
Integer Ambiguity Resolution

Some basic integer solving

$$\hat{a} = [15.23]$$

$$\sigma_{\hat{a}} = 2$$

$$a = ?$$



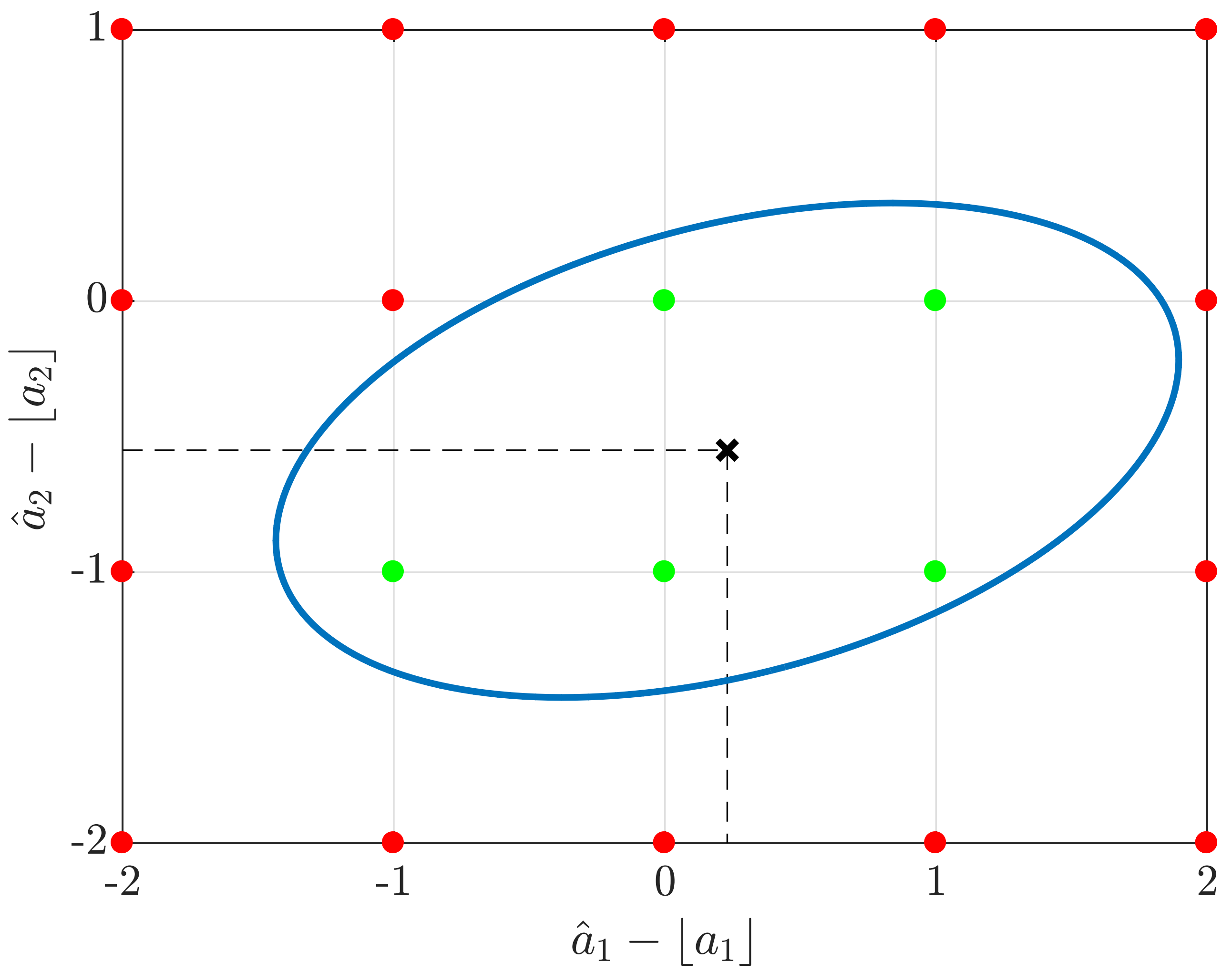
Integer Ambiguity Resolution

Some basic integer solving

$$\hat{a} = \begin{bmatrix} 15.23 \\ -36.55 \end{bmatrix}$$

$$\mathbf{Q}_{\hat{a}} = \begin{bmatrix} 2 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

$$a = ?$$

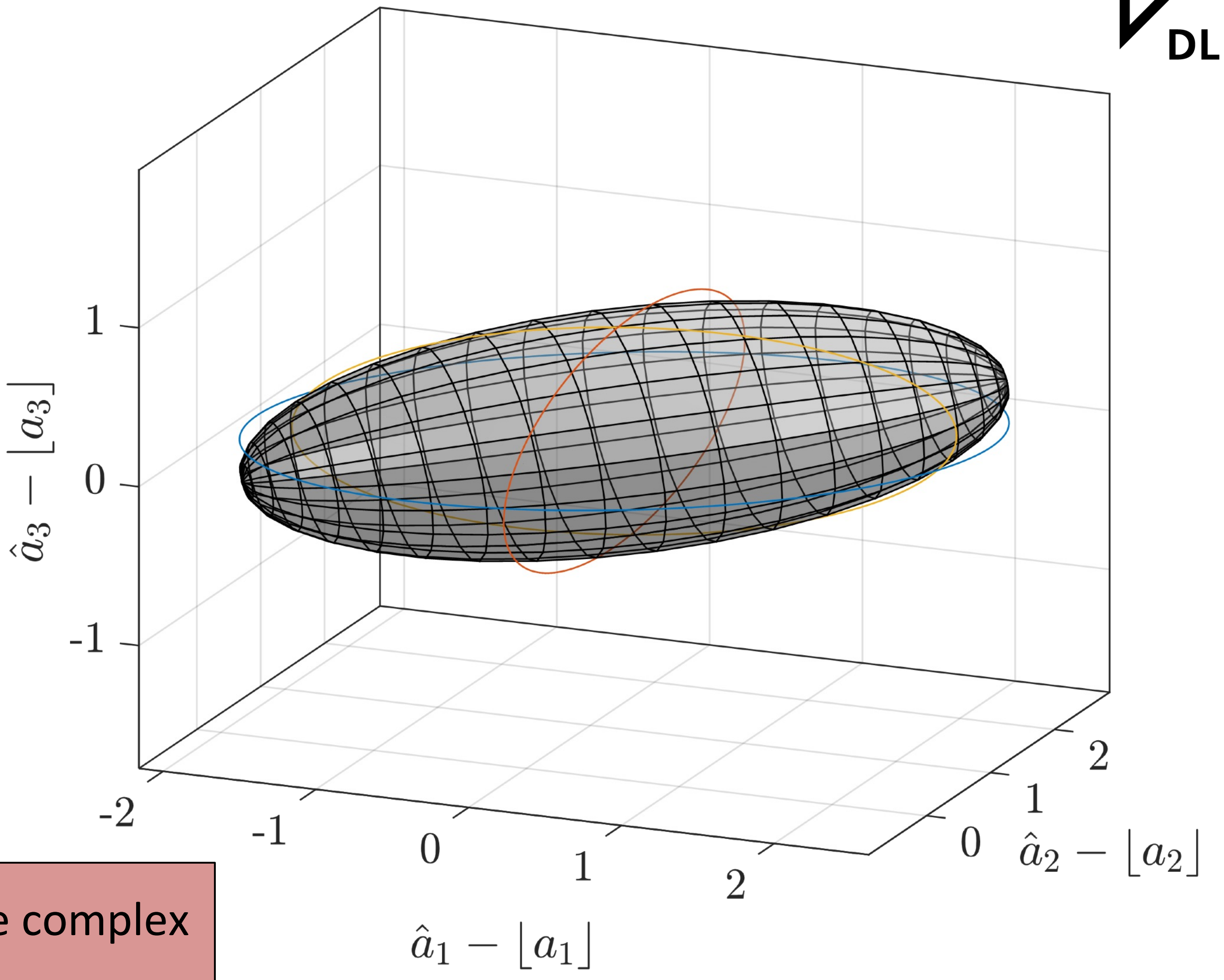


Integer Ambiguity Resolution

Some basic integer solving

$$\hat{a} = \begin{bmatrix} 15.23 \\ -36.55 \\ 44.11 \end{bmatrix}$$

$$\mathbf{Q}_{\hat{a}} = \begin{bmatrix} 2 & 0.5 & 0.2 \\ 0.5 & 1.5 & 0.05 \\ 0.2 & 0.05 & 0.2 \end{bmatrix}$$



Integer ambiguity resolution becomes more complex with the number of observations

Basics on Integrity Monitoring for GNSS

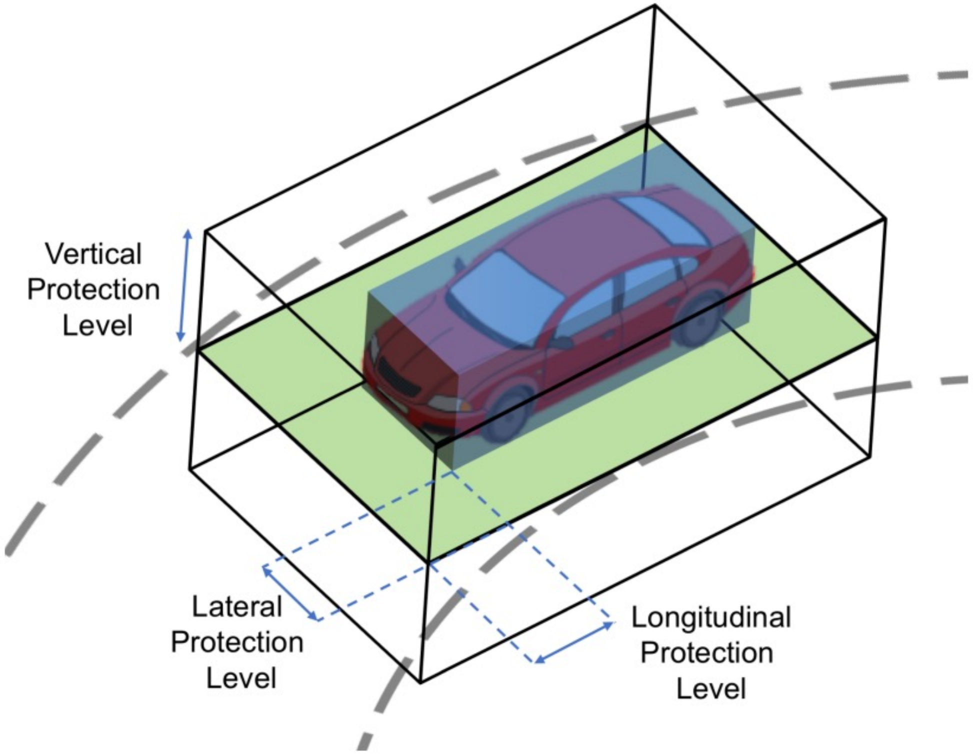
Integrity monitoring measures the trust on the navigation estimates & provides timely warnings when an unacceptable fault occurs / system is unreliable

Navigational requirements

- Accuracy
- Continuity
- Availability

Integrity components

- Alert Limit
- Integrity Risk
- Time to Alert
- Protection Level



Reid, Tyler GR, et al. "Localization requirements for autonomous vehicles." *arXiv preprint arXiv:1906.01061* (2019).

Basics on Integrity Monitoring for GNSS

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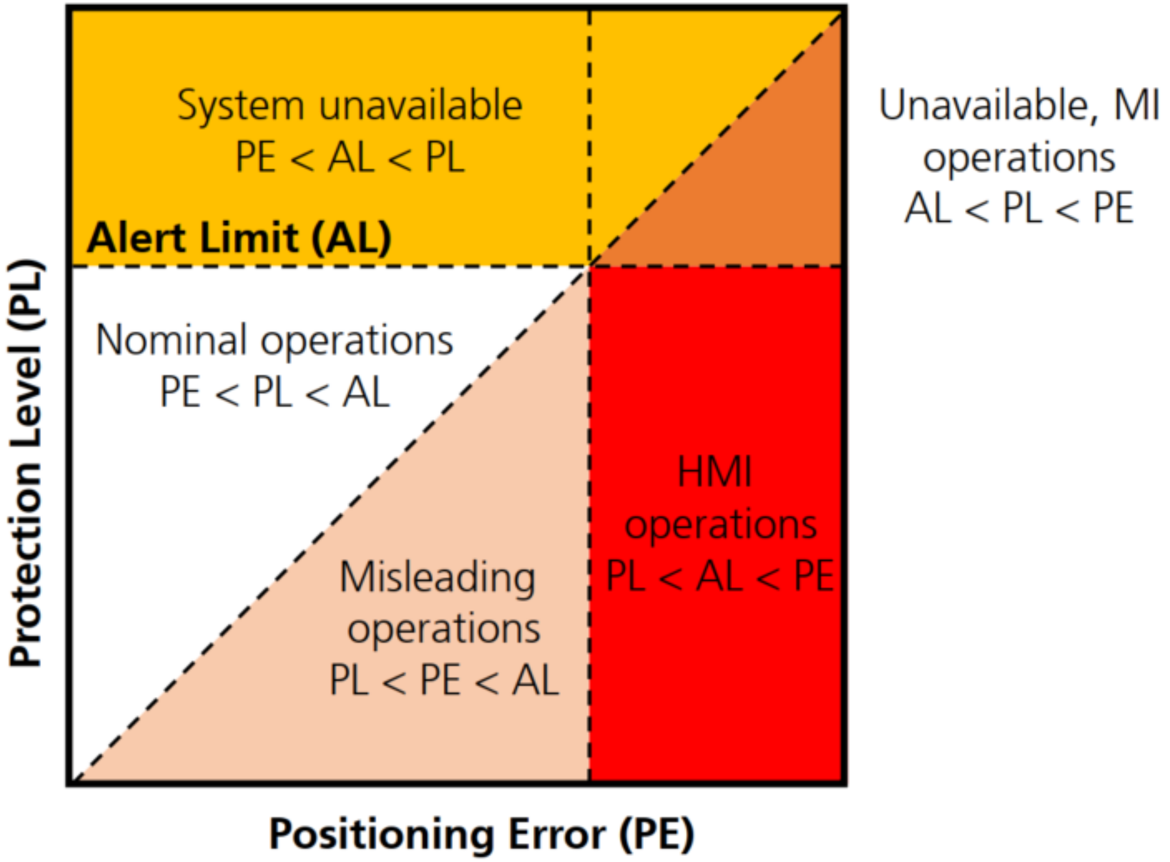


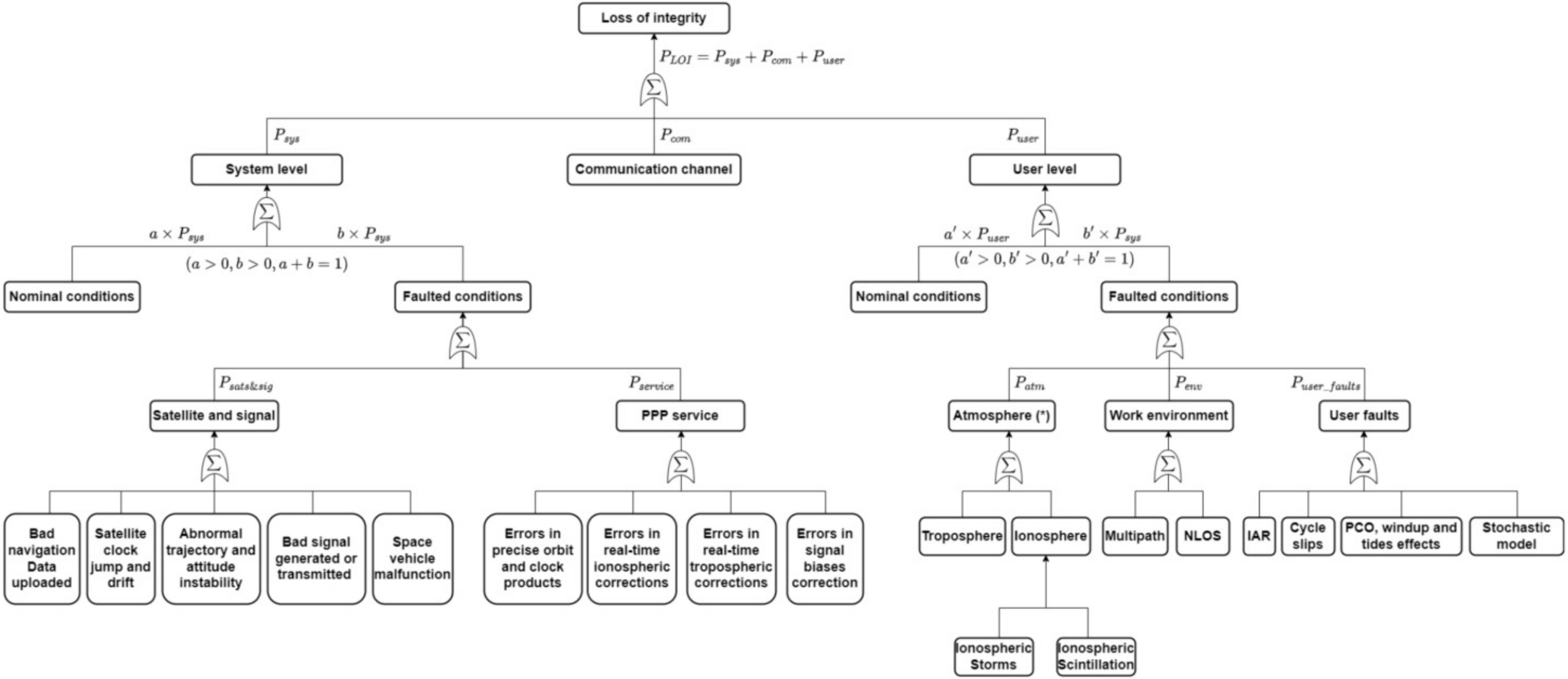
State of the Art on Integrity Monitoring: the limitations

- Standard solutions are derived specifically for aviation purposes:
  - open sky assumption
  - very low number of faults (only due to satellite faults)
  - not applicable to landing / take-off maneuvers
- Typically, only code observations are used (or code-carrier smoothing)
- Only snapshot solutions are considered (no recursive estimation)
- Multi sensor integration and related challenges are not contemplated
- Availability of Integrity Support Message (ISM), meaning “perfect” stochastic modeling

The integrity monitoring community is hard to access and even harder to convince

There is a need for new methods on Integrity Monitoring!

Integrity Threats for Precise Navigation



Loss of integrity

$P_{LOI} = P_{sys} + P_{com} + P_{user}$

P_{sys} System level

$a \times P_{sys}$ $b \times P_{sys}$

$(a > 0, b > 0, a + b = 1)$

Nominal conditions

Faulted conditions

$P_{sats\&sig}$ Satellite and signal

- Bad navigation Data uploaded
- Satellite clock jump and drift
- Abnormal trajectory and attitude instability
- Bad signal generated or transmitted
- Space vehicle malfunction

$P_{service}$ PPP service

- Errors in precise orbit and clock products
- Errors in real-time ionospheric corrections
- Errors in real-time tropospheric corrections
- Errors in signal biases correction

P_{com} Communication channel

P_{user} User level

$a' \times P_{user}$ $b' \times P_{sys}$

$(a' > 0, b' > 0, a' + b' = 1)$

Nominal conditions

Faulted conditions

P_{atm} Atmosphere (*)

- Troposphere
- Ionosphere
  - Ionospheric Storms
  - Ionospheric Scintillation

P_{env} Work environment

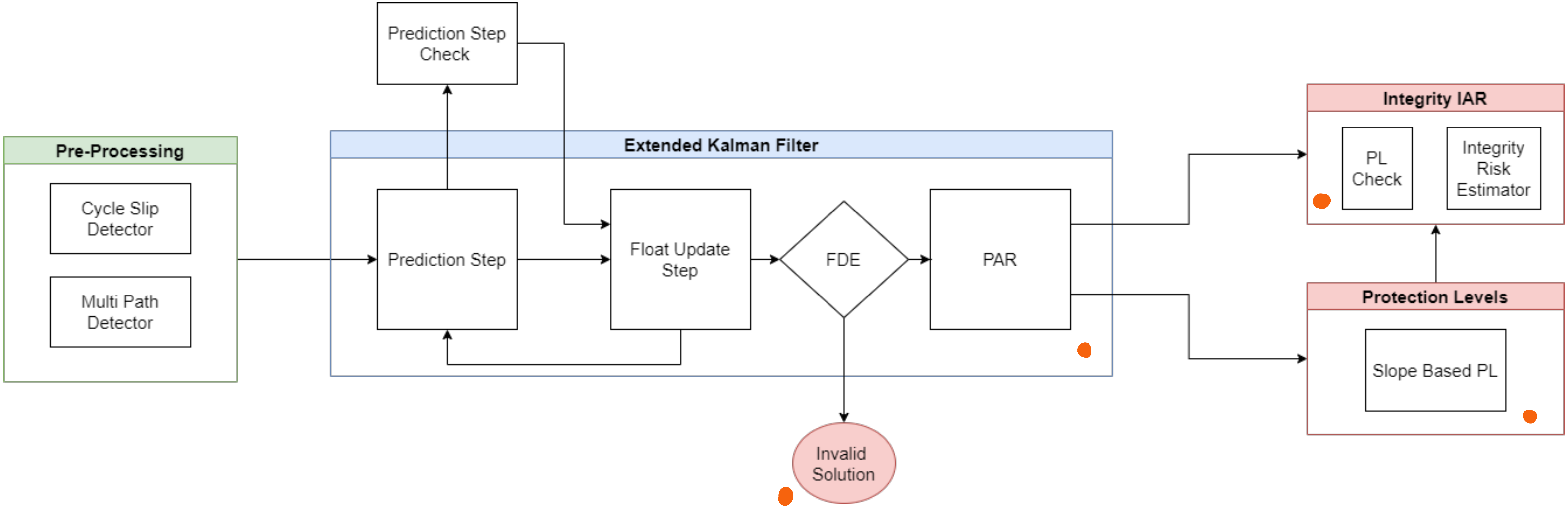
- Multipath
- NLOS

P_{user_faults} User faults

- IAR
- Cycle slips
- PCO, windup and tides effects
- Stochastic model

So… what is the least we could do?

1. **pre-processing**
2. **estimator + Fault Detection and Exclusion (FDE)** mechanism (+ Test statistic)
3. **error bounding** (protection level / integrity risk)



Multi Hypothesis-based Filtering

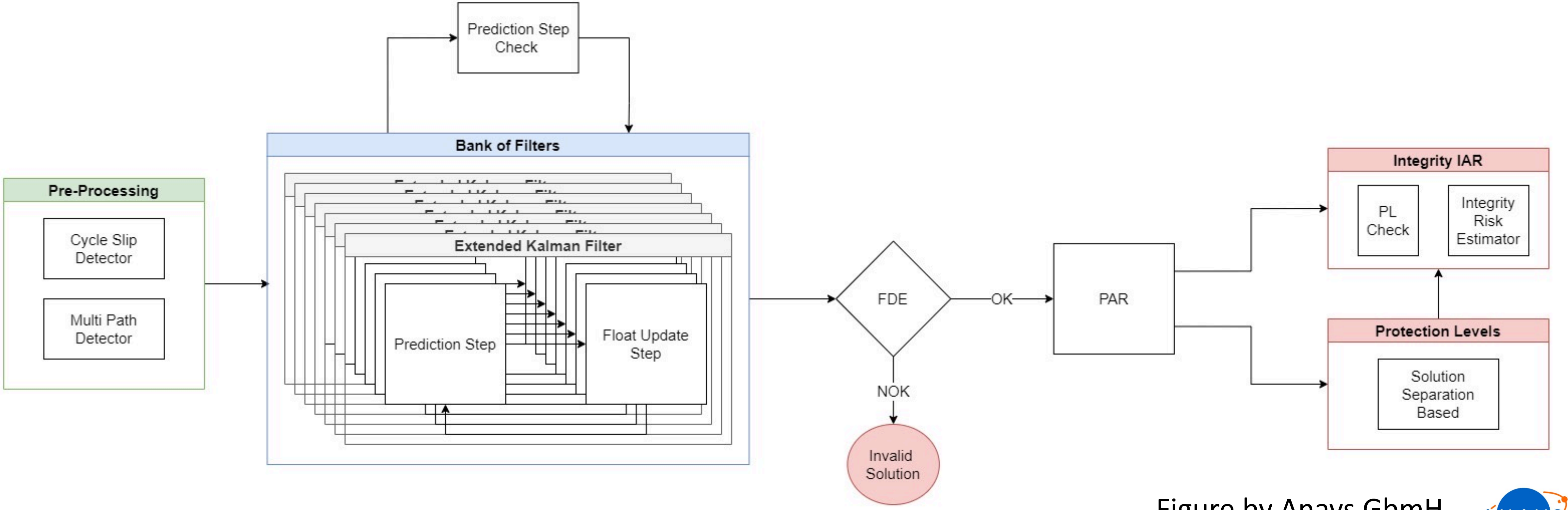


Figure by Anavs GbmH

Pros

- Lower PLs expected
- Fault Detection and Exclusion inherently covered

Cons

- Computational Cost → Operational feasibility?