2-Channel TI-ADC parameters estimation using Periodic Nonuniform Sampling (PNS) formulas

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Abstract

Two parallel Analog-to-Digital Converters (ADC) are the components of 2-Channel TI-ADC (for Time-Interleaved ADC). They are addressed at times nT and $nT+\frac{T}{2}, n\in\mathbb{Z}$ (T is the clock period). "Timing skews" are biases of these sampling times. They vary slowly, and they have to be estimated and corrected. In this article, we give a method of estimation which utilizes results about Periodic Nonuniform Sampling of order 2.

keywords: 2-Channel TI-ADC, periodic nonuniform sampling, sampling formula, timing skew.

1 Introduction

Two-Channel TI-ADC are devices constituted by two ADC in parallel (ADC_1 and ADC_2). The resulting ADC allows better performances than a unique ADC and at lower price ([1] to [7]). The clock of the device addresses each elementary ADC successively at times nT and $\left(nT+\frac{T}{2}\right), n\in\mathbb{Z}$, where T is the clock period. Actually, the circuitry and differences between ADC_1 and ADC_2 introduce a bias $\theta-\frac{T}{2}$ between sampling times. $\theta-\frac{T}{2}$ is a "timing skew". Other error sources are more easily addressed (like offsets and gains).

If $g(t)$ is the input of the device, the output is constituted by sequences

$$\mathbf{g}=\{g(nT), g(nT+\theta), n\in\mathbb{Z}\} \tag{1}$$

for undetermined θ and assuming that the quantification rate is small enough and no bias due to offsets or gains.

In the sampling theory framework, we are in the situation of a PNS2 (order 2 Periodic Nonuniform Sampling). This type of plan is well documented ([8] to [12]). We know that the function (or the stationary random process) $g(t)$ can be recovered provided weak conditions and $\theta\notin T\mathbb{Z}$.

In what follows, we consider a function $g(t), t\in\mathbb{R}$, with a Fourier transform $G(f)$ (the "spectrum" of $g(t)$) such as

$$g(t)=\int_{-1/T}^{1/T} e^{2i\pi ft} G(f)\, df. \tag{2}$$

Because the length support of $G(f)$ is at most $2/T$, the sequence $\mathbf{g}$, sampled at the mean rate $T/2$, brings enough information to reconstitute $g(t)$, provided weak properties about $G(f)$. The simple formula hereafter gives the solution (see Appendix 1):

$$g(t)=-A_0(t)\frac{\sin\pi(t-\theta)/T}{\sin\pi\theta/T}+A_\theta(t)\frac{\sin\pi t/T}{\sin\pi\theta/T} \tag{3}$$

$$A_x(t)=\sum_{n\in\mathbb{Z}}(-1)^n \sin c\pi\left(\frac{t-x}{T}-n\right) g(nT+x) \tag{4}$$

where $\text{sinc}x=(\sin x)/x$. Consequently, when the timing skew θ is known with enough accuracy, (3) and (4) permit an errorless reconstruction $g(t)$ whatever t or only the sequence

$$\mathbf{g}'=\{g(nT/2), n\in\mathbb{Z}\}$$

which is the expected output of the device. In this case, the problem is to find a good estimation of the parameter θ. This parameter can show slow variations with time, for instance due to temperature variations, which implies a constant monitoring of the estimation of θ.

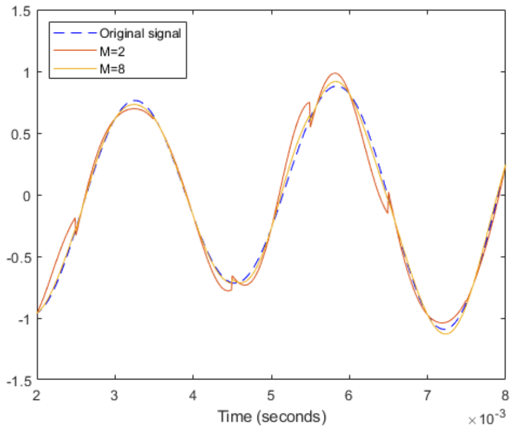

Figure 1: Reconstruction of three mixed pure tones.

It is admitted that the infinite sum (4) does not converge very quickly. Nevertheless, good results are obtained with a few number of terms. In figure 1, $T = 1$, $\theta = 0.58$ and $g(t)$ is the sum of three pure tones at 0.152, 0.336, 0.843. We see that the approximation of (3) is good above M =8 terms (4 for each sum), but surprisingly acceptable for $M = 2$ (the periodical bounces are due to changes of data which are taken into account).

Formula (3) provides a erroneous result when the timing skew $\theta - \frac{T}{2}$ between ADC_1 and ADC_2 is not accurately measured. In the following section, we propose a new method of estimation based on a good use of (3).

2 Timing skew estimation

Let assume that we know an approximate value $\widetilde{\theta}$ of θ. We define $\widetilde{g}(t)$ from $g(t)$ by

$$\widetilde{g}(t) = -A_0(t)\frac{\sin\pi\left(t-\widetilde{\theta}\right)/T}{\sin\pi\widetilde{\theta}/T} + \widetilde{A}_\theta(t)\frac{\sin\pi t/T}{\sin\pi\widetilde{\theta}/T} \tag{5}$$

$$\widetilde{A}_\theta(t) = \sum_{n\in\mathbb{Z}}(-1)^n \sin c\pi\left(\frac{t-\widetilde{\theta}}{T}-n\right)g(nT+\theta). \tag{6}$$

$g(nT+\theta)$ is the output of ADC_2, at a time $nT+\theta$ which is not wellknown. The sequence of data $\mathbf{g}$ of (1) allows to calculate $\widetilde{A}_\theta(t)$ (an approximation depending on the number of used terms in infinite sums). For $g(t) = \cos 2\pi f t$, $f \in \left(0, \frac{1}{T}\right)$, formula (11) in appendix 2 is available. It means that an erroneous value $\widetilde{\theta}$ of θ entered in formula (3) splits a spectral line at $f \in (0, 1/T)$ into two lines, one at f and the second one at $\frac{1}{T} - f$, which remains in $\left(0, \frac{1}{T}\right)$.

frequency	amplitude	phase
$f \in \left(0, \frac{1}{T}\right)$	$\frac{\sin\pi\left(f\delta\theta+\frac{\widetilde{\theta}}{T}\right)}{\sin\frac{\pi\widetilde{\theta}}{T}}$	$-\frac{\delta\theta}{2}$
$\frac{1}{T} - f$	$\frac{-\sin(\pi f\delta\theta)}{\sin\frac{\pi\widetilde{\theta}}{T}}$	$\frac{f\delta\theta+\frac{\widetilde{\theta}}{T}}{2\left(\frac{1}{T}-f\right)}$

Table 1: Amplitude and phase of $\widetilde{\cos}2\pi f t$

Table 2 below gives the amplitudes and the phases of $\widetilde{\cos}2\pi f t$ as functions of $f, \theta, \widetilde{\theta}, \delta\theta = \theta - \overline{\theta}$ (f and $\widetilde{\theta}$ are given but θ is unknown):

In a real two-channel TI-ADC, parameters θ and $\widetilde{\theta}$ are close to $T/2$. Therefore, we see the appearance of a parasite line at $\left(\frac{1}{T} - f\right)$ with amplitude close to $\pi f \delta\theta$. In the same time, the main line amplitude remains around 1. To find a good value of θ, it suffices to vary $\widetilde{\theta}$ in computations of (5) and (6) up to the cancelation of the parasite line.

At first sight, it is beneficial to take f close to $1/T$ for increasing $\pi f\left|\delta\theta\right|$. But to take $\frac{1}{T} - f$ close to 0 makes harder the estimation of the amplitude of the parasite line. To overcome this difficulty, it seems reasonable to work with several spectral lines which generate so much parasite lines in $\left(0, \frac{1}{T}\right)$.

Figure 2 illustrates the situation of figure 1: the input $g(t)$ is a well determined mixing of three pure tones at frequencies 0.152, 0.336, 0.843 (the curve in bold $\mathbf{C}$). Formula (5) is used from $\widetilde{\theta} = 0.50$ using data $g(n+\theta)$ delivered by the device with unknown θ. We see that resulting curves $\mathbf{C}_{\widetilde{\theta}}$ approach $\mathbf{C}$ when we increase $\widetilde{\theta}$ up to a neighbourhood of 0.58. Consequently, the true value θ =0.58 can be well approximated. In parallel, Table 2 shows the decrease of parasite lines amplitude at $1-f = 0.157, 0.664, 0.848$ (from unit amplitudes of input lines). Good values of $\widetilde{\theta}$ minimize the amplitude of these lines.

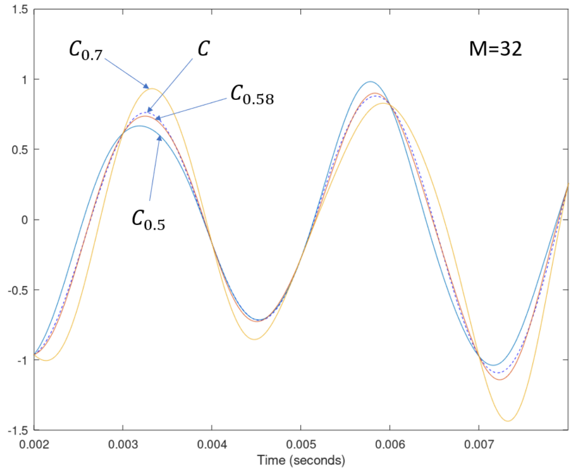

Figure 2: Reconstruction of three mixed pure tones with different values of $\widetilde{\theta}$.

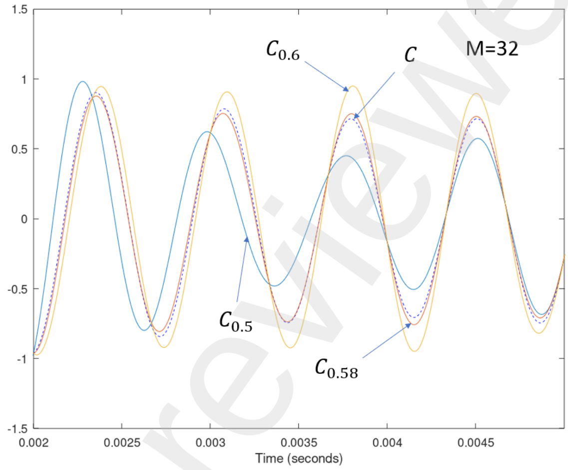

Figure 3: Reconstruction of three mixed pure tones with different values of $\widetilde{\theta}$ in two bands.

$1-f$ \ $\widetilde{\theta}$	0.50	0.54	0.575	0.58	0.585	0.6
0.157	210	110	14	0	14	53
0.664	80	40	5	0	5	21
0.848	40	19	2	0	2	10

Table 2: parasite lines amplitude $\times 10^3$

3 The two-band case

A 2-TI-ADC can work in a two-band context. If (2) is generalized in

$$g(t) = \int_{\Delta_k} e^{2i\pi ft} G(f)\, df \tag{7}$$

$$\Delta_k = \left(\frac{-k-1}{T}, \frac{-k}{T}\right) \cup \left(\frac{k}{T}, \frac{k+1}{T}\right), k \in \mathbb{N}$$

then (5) becomes

$$\widetilde{g}(t) = -A_0(t) \frac{\sin \pi\alpha \left(t - \widetilde{\theta}\right)/T}{\sin \pi\widetilde{\theta}/T} + \widetilde{A}_\theta(t) \frac{\sin \pi\alpha t/T}{\sin \pi\widetilde{\theta}/T} \tag{8}$$

with $\alpha = 2k+1$. Formula (6) is unchanged, $g(t)$ is recovered without error from (8) when $\widetilde{\theta} = \theta$. Intuitively, for a same accuracy, $\delta = \theta - \widetilde{\theta}$ has to decrease with $1/k$. Figure 3 illustrates this property: when $k = 2$, for frequencies 2.152, 2.336, 2.843, $\theta = 0.58, T = 1$, and for the same $\widetilde{\theta}$, the gap between the curves have increased. A good estimation of θ is achieved when the curves $\mathbf{C}_\theta$ and $\mathbf{C}_{\widetilde{\theta}}$ of $g(t)$ and $\widetilde{g}(t)$ are confused.

4 Conclusion

A PNSN (order N Periodic Nonuniform Sampling of) is a sampling plan based on a sampling sequence $\mathbf{t}$ of shape

$$\mathbf{t} = \{nT + t_k, n \in \mathbb{Z}, k = 1, 2, .., N\}.$$

This kind of sampling was studied by J. L. Yen [7] for functions in baseband like (2) (the "Recurrent Nonuniform Sampling", third example in the Yen's paper). Besides, a $N-$Channel TI-ADC is constituted by N elementary ADC in parallel with regular spaces. It corresponds to a PNSN where (ideally)

$$t_k = kT/N, k = 1, 2, .., N.$$

Gaps from these t_k are the "timing skews". They happen most of the time because of differences between elementary ADC and because the circuitry. They may vary in time, and their estimation is the main problem of TI-ADC.

In this paper, we remain in the case $N = 2$, which is sufficient and is easily generalized for any N.

We propose estimations of timing skews $\theta - 1/T$ based on properties of PNS2. Being chosen some $g(t)$ as input of a 2-Channel TI-ADC, the output is the set $\mathbf{g}$ defined by (1). Though θ is not well-known (and may be varying slowly), the $g(nT+\theta)$ are available (they constitute the half-part of the output). Formulas (5) and (6) provide a estimation $\widetilde{g}(t)$ of $g(t)$ which confuses with $g(t)$ when $\widetilde{\theta} = \theta$. Consequently, a routine starting for a near enough value $\widetilde{\theta}$ of θ will converge to a good estimation of θ.

When this estimation is done, formulas (3) and (4) allow an errorless reconstruction of any function (or random process) $g(t)$ with spectra like (2) at any time t and wathever the parameter θ (ideally equal to $T/2$ for a 2-Channel TI-ADC). In particular, good values are obtained at times $nT/2$.

Figures illustrate the proposal, $g(t)$ being a mixing of 3 pure tones and $T = 1, \theta = 0.58$. Figure 1 shows that computations are cheap because few elements are used. Figure 2 gives $g(t)$ (in bold) and computations of $(5),(6)$ with $\widetilde{\theta} = 0.5, 0.58, 0.7$. $\widetilde{g}(t)$ approaches $g(t)$ when $\widetilde{\theta}$ approaches $\theta = 0.58$. Figure 3 is for pure tones shifted in bands $(-2,-1) \cup (1,2)$ (the two-band case).

Finally, this paper proposes an estimation method which uses a test function $g(t)$ as input. The output $\mathbf{g}$ is entered in formulae $(5),(6)$ which provide $\widetilde{g}(t)$ for values $\widetilde{\theta}$ around 0.5, the value linked to the ideal 2-Channel TI-ADC. We obtain θ when $\widetilde{g}(t)$ and $g(t)$ coincide.

When $N > 2$, each timing skew can be estimated, using both ADC. More general formulas are available for reconstruction [8], [13], [14].

5 Appendices

5.1 Appendix 1

The classical sampling formula in its simplest form is

$$e^{2i\pi ft} = \sum_{n\in\mathbb{Z}} \sin c\pi \left(\frac{t}{T} - n\right) e^{2i\pi fnT}, f \in \left(\frac{-1}{2T}, \frac{1}{2T}\right)$$

whatever $t \in \mathbb{R}$. We change f in $f + \frac{1}{2T}$, $f - \frac{1}{2T}$ and t in $t - x$ to obtain, (whatever t, x):

$$\alpha_n(t,x) = \begin{cases} e^{2i\pi ft + i\pi(t-x)/T}, f \in \left(\frac{-1}{T}, 0\right) \\ e^{2i\pi ft - i\pi(t-x)/T}, f \in \left(0, \frac{1}{T}\right) \end{cases}$$

for $\alpha_n(t,x) = \sum_{n\in\mathbb{Z}} (-1)^n \sin c\pi \left(\frac{t-x}{T} - n\right) e^{2i\pi f(nT+x)}$. (9)

These formulas are brought in

$$g_-(t) = \int_{-1/T}^{0} e^{2i\pi ft} G(f)\, df$$

$$g_+(t) = \int_{0}^{1/T} e^{2i\pi ft} G(f)\, df.$$

which leads to

$$g_-(t) = e^{-i\pi(t-x)/T} \sum_{n\in\mathbb{Z}} (-1)^n \sin c\pi \left(\frac{t-x}{T} - n\right) g_-(nT+x)$$

$$g_+(t) = e^{i\pi(t-x)/T} \sum_{n\in\mathbb{Z}} (-1)^n \sin c\pi \left(\frac{t-x}{T} - n\right) g_+(nT+x).$$

We deduce the set of equations (for any x)

$$g_+(t)\, e^{-i\pi(t-x)/T} + g_-(t)\, e^{i\pi(t-x)/T} = A_x(t)$$

$$A_x(t) = \sum_{n\in\mathbb{Z}} (-1)^n \sin c\pi \left(\frac{t-x}{T} - n\right) g(nT+x).$$

Provided that $x \notin T\mathbb{Z}$, we obtain

$$g(t) = -A_0(t) \frac{\sin \pi (t-x)/T}{\sin \pi x/T} + A_x(t) \frac{\sin \pi t/T}{\sin \pi x/T}. \quad (10)$$

5.2 Appendix 2

For $g(t) = e^{2i\pi ft}$, we have, from (6) and (9)

$$\widetilde{A}_\theta(t) = \begin{cases} e^{2i\pi\left(f+\frac{1}{2T}\right)\left(t-\widetilde{\theta}\right)}, f \in \left(\frac{-1}{T}, 0\right) \\ e^{2i\pi\left(f-\frac{1}{2T}\right)\left(t-\widetilde{\theta}\right)}, f \in \left(0, \frac{1}{T}\right) \end{cases}$$

Held in (5), we obtain, when $f \in \left(0, \frac{1}{T}\right)$

$$\widetilde{\cos} 2\pi ft = -\cos\left(2\pi ft - \frac{\pi t}{T}\right) \frac{\sin \frac{\pi\left(t-\widetilde{\theta}\right)}{T}}{\sin \frac{\pi\widetilde{\theta}}{T}}$$

$$+ \cos\left(2\pi f\theta + 2\pi\left(f - \frac{1}{2T}\right)\left(t - \widetilde{\theta}\right)\right) \frac{\sin \frac{\pi t}{T}}{\sin \frac{\pi\widetilde{\theta}}{T}}.$$

Few trinometrical manipulations lead to ($\delta\theta = \theta - \widetilde{\theta}$)

$$\widetilde{\cos} 2\pi f t = \frac{\sin\left(\pi\left(f\delta\theta + \frac{\widetilde{\theta}}{T}\right)\right)}{\sin\frac{\pi\widetilde{\theta}}{T}} \cos\left(2\pi f t + \pi f \delta\theta\right)$$

$$-\frac{\sin \pi f \delta\theta}{\sin\frac{\pi\widetilde{\theta}}{T}} \cos\left(2\pi t\left(\frac{1}{T} - f\right) - \pi\left(f\delta\theta + \frac{\widetilde{\theta}}{T}\right)\right). \quad (11)$$

Figure 1 illustrates this result. Any frequency line at $f \in \left(0, \frac{1}{T}\right)$ suffers a change of amplitude and phase, and a displacement with changes towards the point $\left(\frac{1}{T} - f\right)$.

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