An EM-based multipath interference mitigation in GNSS receivers☆

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A B S T R A C T

In multipath (MP) environments, the received signals depend on several factors related to the global navigation satellite systems (GNSS) receiver environment and motion. Thus it is difficult to use a specific propagation model to accurately capture the dynamics of the MP signal when the GNSS receiver is moving in urban canyons. This paper formulates the problem of MP interference mitigation in the GNSS receiver as a joint state (containing the direct signal parameters) and time-varying model parameter (containing the MP signal parameters) estimation. Accordingly, we propose to exploit the EM algorithm for achieving the joint state and time-varying parameter estimation in the context of MP interference mitigation in GNSS receivers. More precisely, the proposed EM-based MP mitigation approach is decomposed into two iterative steps: (a) the posterior pdf of the direct signal parameters and the expected log-likelihood function necessary in the expectation step of the EM algorithm are approximated by using an appropriate particle filter; (b) the maximum likelihood solution for MP signal parameters is then obtained using Newton's method in the maximization step. The convergence of the proposed approach is analyzed based on the existing convergence theorem associated with the EM algorithm. Finally, a comprehensive simulation study is conducted to compare the performance of the proposed EM-based MP mitigation approach with other state-of-the-art MP mitigation approaches in static and realistic scenarios.

1. Introduction

With the new application requirements for global navigation satellite systems (GNSS) in complex environments, such as in urban canyons, one important remaining challenge is to reduce the impact of multipath (MP) on positioning methods. MP signals are mainly due to the fact that a signal transmitted by a navigation satellite is very likely to be reflected or diffracted and can follow different paths before arriving at the GNSS receiver [1]. In general, there are two classes of perturbations affecting the received GNSS signals: (a) MP interferences resulting from the sum of the direct signal and of delayed reflections handled by the GNSS receiver (b) Non-line of sight (NLOS) signals which result from a unique reflected signal received and tracked by the GNSS receiver [2,3]. In some studies, both MP interference and NLOS signals are often considered as MP, but they are due to different phenomena that cause different ranging errors [4,5]. Although these two phenomena usually arise together in urban canyons, it is clearly interesting to separate them in practical applications.

1.1. Previous work

The GNSS receiver has to track the signal composed of the direct LOS signal possibly affected by delayed reflections in the presence of MP interferences. The correlation function of the LOS signal is distorted by the existing MP signals, and this distortion results in tracking errors leading to biases in code delay and carrier phase estimation [6]. Different MP interference mitigation approaches have been proposed in the literature for mitigating MP interference error within the GNSS receiver. Several narrow correlator delay lock loop (DLL) methods [7] have been proposed in order to eliminate MP interferences by correcting the shape of the discriminator function or the correlation function in the DLL, such as the strobe correlator [8], the early-late-slope technique [9], the double-delta correlator [10]. Based on the fact that the standard discriminator output, which is specifically normalized, has an invariant point in the presence of MP interferences, the MP insensitive DLL proposed in [11] synchronizes the late replica code

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to the direct signal code. Another solution is the coupled amplitude DLL consisting of several parallel tracking units, which implements a feedback loop to separate out the direct and MP signals and then track each signal parameters in parallel units [12].

The parameters of the LOS and reflected signals can be estimated by using a statistical approach based on the maximum likelihood (ML) principle [13], such as the MP estimating delay lock loop (MEDLL) [14] and the vision correlator [15] introduced by NovAtel, which are two different implementations of MP mitigation techniques [16]. However, these approaches need to process the received signal by using a cross-correlation function with multiple correlators and algorithms that can be computationally intensive [17]. Note that a grid search approach has been proposed in [18] allowing a better implementation of the ML estimator via a global maximization approach. In order to reduce the computational complexity of ML-based MP mitigation approaches, Newton's method has been also investigated for iteratively computing the ML estimators of the direct and MP signal parameters [19,20].

Dynamic estimation approaches assume that the time propagation associated with the unknown parameters of direct and MP signals can be modelled by a first-order Markov model, which provides the time-dependent prior probability density function (pdf) for the unknown parameters [21]. The objective is then to estimate recursively the posterior pdf of the unknown parameters associated with the direct and MP signals. Considering that the correlation function is related to the unknown parameters by highly nonlinear equations, the use of a Rao–Blackwellized particle filter (RBPF) has been proposed in the literature to generate samples distributed according to the posterior distribution of interest and estimate the state vector with the other unknown parameters [22]. Some approaches have also been suggested to improve the efficiency of these filters. For instance, a data compression method based on the ML estimation was used to decrease the dimension of the observation vector in order to reduce the complexity of the MP mitigation technique [23]. A deterministic form of a particle filter was proposed in [24] for joint MP detection and navigation parameter estimation with a low number of particles. A two-fold marginalized Bayesian filter was finally proposed in [25] allowing the number of received MP signals and the corresponding signal parameters to be estimated in a track-before-detect fashion.

1.2. Main contributions and paper organization

A prior dynamic equation associated with the time propagation of direct signal parameters (this propagation is related to the vehicle motion) can be defined when the GNSS signal has been locked inside the receiver [26]. An important property of MP signals is that they not only depend on the relative position of the receiver and GNSS satellites, but also on the environment where the receiver is located, especially in urban canyons [27]. Since it is difficult to use a specific propagation model to accurately capture the dynamics of MP signals, we propose in this work to consider the MP signal parameters as unknown time-varying quantities. As a consequence, the point of view considered in this work is to formulate the problem of MP interference mitigation in the GNSS receiver as a joint state (direct signal parameters) and time-varying model parameter (MP signal parameters) estimation problem using a state-space model (i.e., state estimation in the presence of model uncertainty).

The expectation-maximization (EM) algorithm proposed in [28] is an effective way to iteratively compute the ML estimator of unknown parameters in probabilistic models involving latent variables [29]. In the EM algorithm, the model parameters are estimated using two iterative steps denoted as expectation (E) and maximization (M). In the E-step, the model parameters are assumed to be known. Therefore an expected log-likelihood function of the complete data under the joint pdf of the state sequence in a fixed-interval needs to be computed. In the M-step, the ML estimates of the model parameters are computed by maximizing the expected log-likelihood function. Accordingly, the EM algorithm has been effectively proposed to compute the ML estimator of unknown model parameters for general state-space models in a batch manner [30–32]. Moreover, an online EM algorithm was investigated in [33] by constructing sufficient statistics for the unknown model parameters using the state and observation vectors.

The main contribution of this paper is to derive an EM algorithm for joint state and time-varying parameter estimation in the context of MP interference mitigation for GNSS receivers. Different from current applications of the EM algorithm to general state-space models, we propose to implement the EM iteration over each observation interval of the receiver due to the fact that the MP signal parameters are assumed to be time-varying and the corresponding sufficient statistics cannot be available in practice. More precisely, the proposed approach is decomposed into two steps: (a) the posterior pdf of the LOS signal parameters and the expected log-likelihood function necessary in the E-step of the EM algorithm are approximated by using an appropriate particle filter; (b) the ML solution for MP signal parameters is obtained using Newton's method in the M-step. The convergence of the proposed approach is analyzed based on the existing convergence theorem associated with the EM algorithm for general state-space models. Finally, a comprehensive simulation study is conducted to compare the performance of the proposed approach with other state-of-the-art MP mitigation approaches in static and realistic scenarios.

This paper is organized as follows: The GNSS signal models in the presence of MP interference are presented in Section 2. Section 3 studies the joint estimation of direct and MP signal parameters in the frame of the EM algorithm. The corresponding convergence properties of the proposed approach is analyzed in Section 4. The performance of the proposed EM-based MP mitigation approach is evaluated in static and dynamic scenarios in Section 5. Conclusions are finally reported in Section 6.

2. GNSS signal model in the presence of MP

In a pilot channel, the received complex baseband signal associated with GNSS satellites affected by M MP signals can be written as follows [1]

$$\boldsymbol{s}(t)=\sum_{m=0}^{M} a_m c(t-\tau_m)e^{j\varphi_m}+\boldsymbol{\omega}(t) \tag{1}$$

with

$$\frac{\mathrm{d}\varphi_m}{\mathrm{d}t}=2\pi f_m^{\mathrm{d}} \tag{2}$$

where a_m, τ_m, φ_m and f_m^{d} are the amplitude, code delay, carrier phase and Doppler frequency of the mth received signal and the subscript $m=0$ denotes the direct LOS signal, $c(t)$ is the pseudo-random noise (PRN) code associated with the GNSS signal, $\boldsymbol{\omega}(t)$ is a zero mean additive complex Gaussian white noise with variance σ^2. In the receiver, sampled data are collected over an observation interval T_{a} (that is composed of N samples of the GNSS signal, i.e., $T_{\mathrm{a}}=NT_{\mathrm{s}}$ where $T_{\mathrm{s}}=\frac{1}{f_s}$ and f_s is the sampling frequency of the GNSS receiver). It can be assumed that all signal parameters do not change significantly over the observation interval T_{a}. As a consequence, the log-likelihood function of all unknown signal parameters for the kth ($k\in\mathbb{N}$) block of N collected samples can be expressed as follows

$$\begin{aligned} L_{\theta^*} \propto & -\int_{kT_a}\left(\boldsymbol{s}(t)-\sum_{m=0}^{M} a_m c(t-\tau_m) e^{j\varphi_m}\right)^2 \mathrm{d}t \\ \propto & -\int_{kT_a}|\boldsymbol{s}(t)|^2 \mathrm{d}t+\sum_{m=0}^{M} 2\mathcal{R}\{\boldsymbol{\alpha}_m^* \boldsymbol{R}_{\mathrm{SC}}(\tau_m)\} \\ & -\sum_{m=0}^{M} \sum_{n=m+1}^{M} 2\Phi(\tau_n-\tau_m)\mathcal{R}\{\boldsymbol{\alpha}_m \boldsymbol{\alpha}_n^*\}-\sum_{m=0}^{M} T_a|\boldsymbol{\alpha}_m|^2 \end{aligned} \tag{3}$$

with

$$\begin{aligned} & \boldsymbol{\theta}^*=\left(a_0, \ldots, a_M, \tau_0, \ldots, \tau_M, \varphi_0, \ldots, \varphi_M, f_0^{\mathrm{d}}, \ldots, f_m^{\mathrm{d}}\right)^{\mathrm{T}} \\ & \Phi(\tau)=\int_{kT_a} c(t) c(t-\tau) \mathrm{d}t \\ & \boldsymbol{R}_{\mathrm{SC}}(\tau)=\int_{kT_a} \boldsymbol{s}(t) c(t-\tau) \mathrm{d}t \end{aligned} \tag{4}$$

where $\boldsymbol{\alpha}_m = a_m e^{j\varphi_m}$ $(m = 0, \ldots, M)$ denotes the complex amplitude of the received signal, $\mathcal{R}\{\cdot\}$ and $(\cdot)^*$ denote the real part and conjugate of a complex number, $\Phi(\cdot)$ is the correlation function of the locally-generated PRN replicas in the receiver, $\boldsymbol{R}_{\mathrm{SC}}(\cdot)$ is the cross correlation function of the received GNSS signal with the PRN replicas over the observation interval.

In practice, the time propagation of the direct LOS signal parameters $\boldsymbol{x} = \left(a_0, \tau_0, \varphi_0, f_0^{\mathrm{d}}\right)^{\mathrm{T}}$ (this propagation is related to the dynamics of the receiver) can be described by a conditional probability density function (pdf) of a state-space model when the GNSS signal has been locked inside the receiver [34], i.e.,

$$\boldsymbol{x}_k \sim f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) = \mathcal{N}\left(\boldsymbol{F}_{k|k-1}\boldsymbol{x}_{k-1}, \boldsymbol{G}_{k|k-1}\boldsymbol{\Sigma}_{x,k-1}\boldsymbol{G}_{k|k-1}^{\mathrm{T}}\right) \tag{5}$$

where $k = 1, \ldots, \infty$ is the kth observation interval, $f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ is the pdf associated with the direct LOS signal parameter dynamics, $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the multivariate Gaussian pdf with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, $\boldsymbol{F}_{k|k-1}$ and $\boldsymbol{G}_{k|k-1}$ are the transition matrices of the direct LOS signal parameters and the process noises and $\boldsymbol{\Sigma}_{x,k-1}$ is the covariance matrix of the process noise (a zero mean Gaussian white noise). More precisely, the matrices $\boldsymbol{F}_{k|k-1}$, $\boldsymbol{G}_{k|k-1}$ and $\boldsymbol{\Sigma}_{x,k-1}$ can be defined as follows [34]

$$\boldsymbol{F}_{k|k-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \lambda T_{\mathrm{a}} \\ 0 & 0 & 1 & T_{\mathrm{a}} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \boldsymbol{G}_{k|k-1} = \begin{pmatrix} T_{\mathrm{a}} & 0 & 0 \\ 0 & T_{\mathrm{a}} & 0 \\ 0 & 0 & \frac{T_{\mathrm{a}}^2}{2} \\ 0 & 0 & T_{\mathrm{a}} \end{pmatrix} \text{and}$$

$$\boldsymbol{\Sigma}_{x,k-1} = \begin{pmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_\tau^2 & 0 \\ 0 & 0 & \sigma_f^2 \end{pmatrix} \tag{6}$$

where $\lambda = f_{\mathrm{co}}/f_{\mathrm{ca}}$ is a scale factor converting the carrier Doppler frequency to the code Doppler frequency, f_{co} and f_{ca} are the PRN code and the GNSS signal carrier frequencies, respectively.

An important property of MP signals is that they not only depend on the relative position of the receiver and GNSS satellites, but also on the environment where the receiver is located, especially in urban canyons. Since it is difficult to use a specific propagation model to accurately capture the dynamics of MP signals, we propose in this work to consider the MP signal parameters as an unknown time-varying model parameter vector, i.e., $\boldsymbol{\theta}_k = \left(\boldsymbol{\theta}_{1,k}, \ldots, \boldsymbol{\theta}_{M,k}\right)^{\mathrm{T}}$ where $\boldsymbol{\theta}_{m,k} = \left(a_{m,k}, \tau_{m,k}, \varphi_{m,k}, f_{m,k}^{\mathrm{d}}\right)^{\mathrm{T}}$ and $m = 1, \ldots, M$. Accordingly, the likelihood function in (3) can be defined using the following conditional pdf

$$\boldsymbol{y}_k \sim g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k) \tag{7}$$

where $\boldsymbol{y}_k = \left(\boldsymbol{s}_{(k-1)T_{\mathrm{a}}+T_{\mathrm{s}}}, \ldots, \boldsymbol{s}_{(k-1)T_{\mathrm{a}}+NT_{\mathrm{s}}}\right)^{\mathrm{T}}$ is the sampled GNSS signal vector over the kth observation interval in GNSS receiver, $g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k)$ is the pdf associated with the observed measurements depending on the unknown model parameter vector $\boldsymbol{\theta}_k$ over the kth observation interval. Note that $g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k)$ can be easily obtained by considering the direct LOS and MP signal parameters in (3) as the state vector $\boldsymbol{x}_k$ and unknown time-varying model parameter vector $\boldsymbol{\theta}_k$ respectively.

3. The EM-based MP interference mitigation in the GNSS receiver

According to (5) and (7), the estimation of the state vector (containing the direct LOS signal parameters) results in a nonlinear filtering problem that can be solved using, e.g., the extended Kalman filter (KF) or the particle filter (PF) in the absence of MP interferences. It has been recognized that the KF and PF-based GNSS signal tracking loops can be possibly implemented in the GNSS receiver [35,36]. However, in the presence of MP interferences, the statistical models used to solve the MP mitigation problem in the GNSS receiver depend on unknown time-varying model parameter vectors (containing the MP signal parameters) that need to be estimated jointly with the state vector. Thus we propose in this work to investigate an EM algorithm over one observation interval for achieving joint state and time-varying parameter estimation in the context of MP mitigation in GNSS receivers (as explained in the subsequent sections).

3.1. Expectation-maximization for MP interference mitigation in GNSS receivers

Since the sampled GNSS signal vectors $\boldsymbol{y}_{1:k} = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_k\}$ over each observation interval are mutually independent, the log-likelihood function $L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k})$ for a sequence of k observation intervals with unknown model parameter vectors $\boldsymbol{\theta}_{1:k} = \left\{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_k\right\} \in \Theta$ (Θ is the feasible set of parameters) can be defined as

$$\begin{aligned} L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}) : & = \log p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}) \\ & = \sum_k \log p_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k) \end{aligned} \tag{8}$$

where $k = 1, \ldots, \infty$ denotes the kth observation interval. Thus the ML estimator $\widehat{\boldsymbol{\theta}}_k^{\mathrm{ML}}$ for the kth observation interval can be obtained by maximizing the corresponding log-likelihood $L_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k) := \log p_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k)$, i.e.,

$$\widehat{\boldsymbol{\theta}}_k^{\mathrm{ML}} = \underset{\boldsymbol{\theta}_k \in \Theta}{\operatorname{argmax}}\, L_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k). \tag{9}$$

The key idea of the EM algorithm is to consider a sequence of k observations $\boldsymbol{y}_{1:k}$ as incomplete data and construct a complete data log-likelihood function for the unknown model parameter vectors $\boldsymbol{\theta}_{1:k}$ by introducing latent state vectors $\boldsymbol{x}_{1:k}$ [33], i.e.,

$$L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k}) := \log p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k}) \tag{10}$$

where $p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k})$ is the joint pdf of the observations and states. Since the EM algorithm iteratively estimates $\boldsymbol{\theta}_{1:k}$ by maximizing the expected log-likelihood of the complete data $L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k})$, the corresponding expected log-likelihood for a sequence of k observation intervals can be defined as follows [30]

$$\begin{aligned} \mathcal{Q}\left(\boldsymbol{\theta}_{1:k}, \widehat{\boldsymbol{\theta}}_{1:k}(r)\right) := & \mathbb{E}_{\widehat{\boldsymbol{\theta}}_{1:k}(r)}\left[L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k})|\boldsymbol{y}_{1:k}\right] \\ = & \int \left[\log p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k})\right] p_{\widehat{\boldsymbol{\theta}}_{1:k}(r)}(\boldsymbol{x}_{1:k}|\boldsymbol{y}_{1:k}) \mathrm{d}\boldsymbol{x}_{1:k} \end{aligned} \tag{11}$$

where $\widehat{\boldsymbol{\theta}}_{1:k}(r)$ is the estimator of the unknown model parameter vectors at the rth EM iteration for a sequence of k observation intervals.

Considering that the pdf associated with the state dynamics in (5) is independent of the unknown model parameter vector $\boldsymbol{\theta}$ over each observation interval, $p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k})$ can be decomposed using the Markov property associated with the state and observation equations (5) and (7)

$$p_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k}) = \prod_k g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k) f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \tag{12}$$

where $k = 1, \ldots, \infty$. After replacing (12) into (10), we obtain

$$\begin{aligned} L_{\boldsymbol{\theta}_{1:k}}(\boldsymbol{y}_{1:k}, \boldsymbol{x}_{1:k}) &= \sum_k \left[\log g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k) + \log f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})\right] \\ &\propto \sum_k \log g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k). \end{aligned} \tag{13}$$

As a consequence, using (13) into (11) leads to

$$\begin{aligned} \mathcal{Q}\left(\boldsymbol{\theta}_{1:k}, \widehat{\boldsymbol{\theta}}_{1:k}(r)\right) &\propto \sum_k \int \log\left[g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k)\right] p_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) \mathrm{d}\boldsymbol{x}_k \\ &= \sum_k \mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r)\right) \end{aligned} \tag{14}$$

where

$$\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r)\right) := \int \log\left[g_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k)\right] p_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) \mathrm{d}\boldsymbol{x}_k \tag{15}$$

and where $\widehat{\boldsymbol{\theta}}_k(r)$ is the estimator of the unknown model parameter vector at the rth EM iteration for the kth observation interval, $p(\boldsymbol{x}_k|\boldsymbol{y}_{1:k})$ is the posterior pdf of the state vector $\boldsymbol{x}_k$ given all available observations $\boldsymbol{y}_{1:k}$ and is referred to as the filtering pdf. According to (14), the expected log-likelihood function $\mathcal{Q}\left(\boldsymbol{\theta}_{1:k}, \widehat{\boldsymbol{\theta}}_{1:k}\right)$ for a sequence of observation intervals is proportional to a summation of the expected log-likelihood function $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ for each observation interval when the pdf associated with the state dynamics is known. Thus the iterative EM solution for a sequence of observation intervals can be decomposed into the corresponding iterative solutions for each observation sequence. In addition, the algorithm for the kth observation interval is such that $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r)\right) > \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right)$ guaranteeing an increase of the log-likelihood $L_{\boldsymbol{\theta}_k}(\boldsymbol{y}_k) > L_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{y}_k)$ at the $r+1$th iteration (see Corollary 1). After choosing some initial value of $\boldsymbol{\theta}$ denoted as $\widehat{\boldsymbol{\theta}}_k(0) \in \Theta$, the EM algorithm generates iteratively a sequence of estimates $\widehat{\boldsymbol{\theta}}_k(r)$ $(r = 0, 1, 2\ldots)$ whose final value approximates the ML estimator of $\boldsymbol{\theta}$ in (9). The EM iteration for MP mitigation in GNSS receivers is summarized in Algorithm 1.

Algorithm 1 The iterative solution of the EM algorithm for an observation interval.

1: **E-Step**. Compute $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r)\right)$

2: **M-Step**. Compute $\widehat{\boldsymbol{\theta}}_k(r+1) = \arg\max_{\boldsymbol{\theta}_k \in \Theta} \mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r)\right)$

3.2. E-step: computing $\mathcal{Q}\left(\theta_k, \widehat{\theta}_k\right)$ based on a particle filter

According to (15), computing $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ in the E-step requires to evaluate the expectation under the posterior pdf $p_{\widehat{\boldsymbol{\theta}}_k}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k})$, where $\widehat{\boldsymbol{\theta}}_k$ has been estimated within the previous M-step and therefore is known. According to the Bayesian estimation principle, the posterior pdf $p_{\widehat{\boldsymbol{\theta}}_k}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k})$ of the state vector $\boldsymbol{x}_k$ (containing the direct signal parameters) given all available observations $\boldsymbol{y}_{1:k}$ can be recursively updated as follows

$$p_{\widehat{\boldsymbol{\theta}}_k}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) \propto p_{\widehat{\boldsymbol{\theta}}_k}(\boldsymbol{y}_k|\boldsymbol{x}_k) p(\boldsymbol{x}_k|\boldsymbol{y}_{1:k-1}) \tag{16}$$

with

$$p(\boldsymbol{x}_k|\boldsymbol{y}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) p_{\widehat{\boldsymbol{\theta}}_{k-1}}(\boldsymbol{x}_{k-1}|\boldsymbol{y}_{1:k-1}) \mathrm{d}\boldsymbol{x}_k \tag{17}$$

where (17) represents a prediction step resulting in the prior pdf of the state vector for the kth observation interval. According to (7), the observation related to the direct LOS signal parameter vector is defined by a highly non-linear equation. Thus, it is difficult to obtain an analytic solution of the posterior pdf in (16). In this situation, it is classical to consider a particle filter approximating the posterior distribution of interest by using a set of weighted particles leading to [37]

$$p_{\widehat{\boldsymbol{\theta}}_k}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_s} \omega_k^i \delta\left(\boldsymbol{x}_k - \boldsymbol{x}_k^i\right) \tag{18}$$

where N_s is the number of particles, $\delta(\cdot)$ is the Dirac delta function, $\boldsymbol{x}_k^i$ is the ith state particle, ω_k^i is an appropriate weight associated with the ith particle and $\sum_{i=1}^{N_s} \omega_k^i = 1$ for the kth observation interval.

As a consequence, $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ in the E-step is approximated by replacing (18) into (15), i.e.,

$$\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right) \approx \widehat{\mathcal{Q}}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right) = \sum_{i=1}^{N_s} \omega_k^i \log g_{\boldsymbol{\theta}_k}\left(\boldsymbol{y}_k|\boldsymbol{x}_k^i\right) \tag{19}$$

where the approximation quality can be enhanced by increasing the number of particles N_s. According to the literature, many particle filter methods have been investigated for approximating the posterior pdf $p(\boldsymbol{x}_k|\boldsymbol{y}_{1:k})$ [38]. All these methods can be used in order to obtain (19).

3.3. M-step: maximizing $\widehat{\mathcal{Q}}\left(\theta_k, \widehat{\theta}_k\right)$ using Newton's method

In the M-step, the approximation $\widehat{\mathcal{Q}}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ needs to be maximized with respective to $\boldsymbol{\theta}_k$ in order to obtain a new iterative ML estimate of the model parameter vector (containing the MP signal parameters) for the kth observation interval. Assuming that a set of weighted state particle $\left\{\omega_k^i, \boldsymbol{x}_k^i\right\}_{i=1}^{N_s}$ have been obtained in the E-step, (19) can be maximized by setting the partial derivatives of $\widehat{\mathcal{Q}}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ with respective to $\boldsymbol{\theta}_k$ to zero, i.e.,

$$\frac{\partial \widehat{\mathcal{Q}}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)}{\partial \boldsymbol{\theta}_k} = \sum_{i=1}^{N_s} \omega_k^i \frac{\partial \log g_{\boldsymbol{\theta}_k}\left(\boldsymbol{y}_k|\boldsymbol{x}_k^i\right)}{\partial \boldsymbol{\theta}_k} = 0 \tag{20}$$

where $\boldsymbol{\theta}_k = \left(\boldsymbol{\theta}_{1,k}, \ldots, \boldsymbol{\theta}_{M,k}\right)^{\mathrm{T}}$, $\boldsymbol{x}_k^i = \left(a_{0,k}^i, \tau_{0,k}^i, \varphi_{0,k}^i, f_{0,k}^{\mathrm{d},i}\right)^{\mathrm{T}}$ is the ith particle associated with the direct LOS signal parameters. Considering that the complex amplitude $\boldsymbol{\alpha}$ in (3) contains the corresponding amplitude and carrier phase of the received signal, (20) leads to

$$\sum_{i=1}^{N_s} \omega_k^i \frac{\partial \log g_{\boldsymbol{\theta}_k}\left(\boldsymbol{y}_k|\boldsymbol{x}_k^i\right)}{\partial \boldsymbol{\alpha}_{m,k}} = 0 \tag{21}$$

$$\sum_{i=1}^{N_s} \omega_k^i \frac{\partial \log g_{\boldsymbol{\theta}_k}\left(\boldsymbol{y}_k|\boldsymbol{x}_k^i\right)}{\partial \tau_{m,k}} = 0 \tag{22}$$

where $m = 1, \ldots, M$.

After replacing (3) into (21), the ML solution of the complex amplitude for the mth MP signal can be expressed as

$$\begin{aligned} \boldsymbol{R}_{\mathrm{SC}}\left(\tau_{m,k}\right) - \sum_{i=1}^{N_s} \omega_k^i \Phi\left(\tau_{m,k} - \tau_{0,k}^i\right) \boldsymbol{\alpha}_{0,k}^i - \sum_{n=1, n\neq m}^{M} \Phi\left(\tau_{n,k} - \tau_{m,k}\right) \boldsymbol{\alpha}_{n,k} \\ - T_a \boldsymbol{\alpha}_{m,k} = 0. \end{aligned} \tag{23}$$

Accordingly, a bank of partial derivatives with respect to $\boldsymbol{\alpha}_m$ ($m = 1, \ldots, M$) can be gathered into the following compact expression

$$\boldsymbol{A}_k \boldsymbol{\alpha}_k = \boldsymbol{b}_k \tag{24}$$

with

$$\boldsymbol{A}_k = \begin{pmatrix} T_a & \Phi(\tau_{2,k} - \tau_{1,k}) & \ldots & \Phi(\tau_{M,k} - \tau_{1,k}) \\ \Phi(\tau_{1,k} - \tau_{2,k}) & T_a & \ldots & \Phi(\tau_{M,k} - \tau_{2,k}) \\ \vdots & \ldots & \ddots & \vdots \\ \Phi(\tau_{1,k} - \tau_{M,k}) & \Phi(\tau_{2,k} - \tau_{M,k}) & \ldots & T_a \end{pmatrix} \tag{25}$$

and

$$\boldsymbol{b}_k = \left(\boldsymbol{b}_{1,k}, \ldots, \boldsymbol{b}_{M,k}\right)^{\mathrm{T}} \tag{26}$$

where $\boldsymbol{b}_{m,k} = \boldsymbol{R}_{\mathrm{SC}}(\tau_{m,k}) - \sum_{i=1}^{N_s} \omega_k^i \Phi(\tau_{m,k} - \tau_{0,k}^i) \boldsymbol{\alpha}_{0,k}^i$ and $m = 1, \ldots, M$. As a consequence, the complex amplitudes of MP signals can be solved in closed form as follows

$$\widehat{\boldsymbol{\alpha}}_k = \left(\boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{A}_k\right)^{-1} \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{b}_k \tag{27}$$

where $\widehat{\boldsymbol{\alpha}}_k = \left(\widehat{\boldsymbol{\alpha}}_{1,k}, \ldots, \widehat{\boldsymbol{\alpha}}_{M,k}\right)^{\mathrm{T}}$. Therefore, the corresponding amplitude and carrier phase of MP signals can be extracted from the estimated complex amplitude $\widehat{\boldsymbol{\alpha}}_k$, i.e.,

$$\begin{aligned} \widehat{a}_{m,k} &= \|\widehat{\boldsymbol{\alpha}}_{m,k}\| \\ \widehat{\varphi}_{m,k} &= \angle \widehat{\boldsymbol{\alpha}}_{m,k} \end{aligned} \tag{28}$$

where $m = 1, \ldots, M$. Since it is difficult to separate the carrier Doppler frequency from the correlation measurements in (3), the carrier Doppler frequency of MP signals is extracted from two successive carrier phase estimation as follows

$$\widehat{f}_{m,k}^{\mathrm{d}} = \frac{\widehat{\varphi}_{m,k} - \widehat{\varphi}_{m,k-1}}{T_{\mathrm{a}}} \tag{29}$$

where $m = 1, \ldots, M$.

Regarding the ML solution of the code delays, replacing (3) into (22) yields

$$\mathcal{R}\left\{ \boldsymbol{\alpha}_{m,k}^* \frac{\partial \boldsymbol{R}_{\mathrm{d}}(\tau_{m,k})}{\partial \tau_m} \right\} = 0 \tag{30}$$

with

$$\begin{aligned} \boldsymbol{R}_{\mathrm{d}}(\tau_{m,k}) &= \boldsymbol{R}_{\mathrm{SC}}(\tau_{m,k}) - \sum_{i=1}^{N_s} \omega_k^i \Phi(\tau_{m,k} - \tau_{0,k}^i) \boldsymbol{\alpha}_{0,k}^i \\ &\quad - \sum_{n=1, n\neq m}^{M} \Phi(\tau_{n,k} - \tau_{m,k}) \boldsymbol{\alpha}_n \\ &= \int_{kT_0} \left[\boldsymbol{s}(t) - \sum_{i=1}^{N_s} \omega_k^i \boldsymbol{\alpha}_0^i c(t - \tau_0^i) - \sum_{n=1, n\neq m}^{M} \boldsymbol{\alpha}_n c(t - \tau_n) \right] \\ &\quad c(t - \tau_m) \mathrm{d}t \end{aligned} \tag{31}$$

where $m = 1, \ldots, M$. Considering that the correlation function in (31) depends on the code delay parameter though the PRN code $c(t - \tau)$, there is no closed form expression for the parameter $\tau_{m,k}$. We propose to use Newton's method in [20] to iteratively compute the ML estimator of the code delay. Thus the iterative solution of the code delay of the mth MP signal is

$$\widehat{\tau}_{m,k}(r+1) = \widehat{\tau}_{m,k}(r) - \frac{\mathcal{R}\left\{ \boldsymbol{R}_{\mathrm{d}}^*(\widehat{\tau}_{m,k}(r)) \frac{\partial \boldsymbol{R}_{\mathrm{d}}(\widehat{\tau}_{m,k}(r))}{\partial \tau_m} \right\}}{\mathcal{R}\left\{ \boldsymbol{R}_{\mathrm{d}}^*(\widehat{\tau}_{m,k}(r)) \frac{\partial^2 \boldsymbol{R}_{\mathrm{d}}(\widehat{\tau}_{m,k}(r))}{\partial \tau_m^2} \right\}} \tag{32}$$

where $m = 1, \ldots, M$ and $r = 0, 1, 2 \ldots$ is the number of iterations. More details about the derivation of the derivatives $\frac{\partial \boldsymbol{R}(\tau)}{\partial \tau}$ and $\frac{\partial^2 \boldsymbol{R}(\tau)}{\partial^2 \tau}$ are given in [20].

Note that the estimated complex amplitude in (27) is a function of the code delay of MP signals for the kth observation interval, whereas the code delay estimator in (32) requires to know the complex amplitude of MP signals. Therefore, the amplitude estimates in (27) at the $(r+1)$th iteration are calculated by using the code delay estimates at the rth iteration. Then the code delay estimates at the $(r+1)$th iteration can be implemented based on the last estimates of complex amplitudes.

Considering that MP signals depend on the environment where the receiver is located, it is difficult to accurately obtain the initial values of the MP signal parameters at the beginning of each EM iteration. Generally, MP signals do not affect positioning results inside the receiver when the code delay offsets of the MP signals with respect to the direct LOS signal are equal or larger than $2T_{\mathrm{c}}$ (where T_{c} is the chip duration of the PRN code). Thus, the estimation of the MP signal parameters is only performed when $\tau_m \in (\tau_0, \tau_0 + 2T_{\mathrm{c}})$ where $m = 1, \ldots, M$. In this paper, the code delays of the MP signals are supposed to be random values uniformly distributed in the interval $\tau_m \in (\tau_0, \tau_0 + 2T_{\mathrm{c}})$ with the condition $\tau_0 < \tau_1 < \ldots < \tau_M < \tau_0 + 2T_{\mathrm{c}}$ at the beginning of the EM iterations for each observation interval. Finally, the EM-based MP interference mitigation approach in GNSS receivers is summarized in Algorithm 2.

Algorithm 2 Proposed EM-based MP interference mitigation approach in GNSS receivers.

Step 1: Initialization ($r = 0$).
1: Calculate $\boldsymbol{x}_k^i \sim f(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}^i)$ according to (5) where $\boldsymbol{x}_k^i = \left(a_{0,k}^i, \tau_{0,k}^i, \varphi_{0,k}^i, f_{0,k}^{\mathrm{d},i}\right)^{\mathrm{T}}$
and $i = 1, \ldots, N_{\mathrm{s}}$.
2: Generate the initial code delay of the mth MP signal $\widehat{\tau}_{m,k}(0) \sim \mathrm{U}(\tau_{0,k}, \tau_{0,k} + 2T_{\mathrm{c}})$
where $\tau_{0,k} = \max_{1<i<N_s} \{\tau_{0,k}^i\}$ and $\widehat{\tau}_{1,k}(0) < \ldots < \widehat{\tau}_{M,k}(0)$ and $\mathrm{U}(a, b)$ denotes the
uniform distribution on the interval (a, b).
3: Calculate the corresponding complex amplitude $\widehat{\boldsymbol{\alpha}}_{m,k}(0)$ for $m = 1, \ldots, M$
using (27).
Step 2: Iteration. **For** $r = 1, 2, \ldots$
% E-step
4: Using the model parameter vector $\widehat{\boldsymbol{\theta}}_k(r-1)$, update each particle weight ω_k^i
based on a particle filter approach (e.g., the bootstrap particle filter [38]) and
compute $\widehat{\mathcal{Q}}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k(r-1)\right)$ according to (19).
% M-step
5: Using the ML estimate of the code delay $\tau_{m,k}(r-1)$ where $m = 1, \ldots, M$,
compute the complex amplitude $\widehat{\boldsymbol{\alpha}}_k(r)$ of MP signals according to (27).
6 Extract the amplitude, carrier phase and Doppler frequency associated with
MP signal according to (28) and (29).
7: Using the ML estimate of the complex amplitude $\widehat{\boldsymbol{\alpha}}_k(r)$, compute the code delay
of the MP signal $\tau_{m,k}(r)$ according to (32) for $m = 1, \ldots, M$.
% Stopping rule
8: If $\frac{\left|\widehat{\mathcal{Q}}(\widehat{\boldsymbol{\theta}}_{r+1}, \widehat{\boldsymbol{\theta}}_r) - \widehat{\mathcal{Q}}(\widehat{\boldsymbol{\theta}}_r, \widehat{\boldsymbol{\theta}}_r)\right|}{\left|\widehat{\mathcal{Q}}(\widehat{\boldsymbol{\theta}}_{r+1}, \widehat{\boldsymbol{\theta}}_r)\right|} < \delta$ or $r \geq r_{\max}$ where $0 < \delta \ll 1$, stop the EM iteration
else set $r = r + 1$.
Step 3: Recursion ($k = k + 1$). Go to Step 1.

4. Convergence analysis

Since it is difficult to establish convergence of the sequence of estimates $\widehat{\boldsymbol{\theta}}(r)$ $(r = 0, 1, 2 \ldots)$, it is classical to prove that the iterative solution of the EM algoritm verifies $\mathcal{Q}\left(\widehat{\boldsymbol{\theta}}(r+1), \widehat{\boldsymbol{\theta}}(r)\right) > \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}(r), \widehat{\boldsymbol{\theta}}(r)\right)$ guaranteeing a strict increase of the log-likelihood $L_{\widehat{\boldsymbol{\theta}}(r+1)}(\boldsymbol{y}_{1:k}) > L_{\widehat{\boldsymbol{\theta}}(r)}(\boldsymbol{y}_{1:k})$ for a sequence of observation intervals at the $(r+1)$th iteration, as demonstrated in Theorem 2 of [39]. In the context of MP mitigation in GNSS receivers, this non-decreasing property over an observation can be obtained based on Theorem 2 of [39].

Corollary 1. *Let $\widehat{\theta}_k(r+1)$ be generated from $\widehat{\theta}_k(r)$ by an iteration of Algorithm 1. Then*

$$L_{\widehat{\boldsymbol{\theta}}_k(r+1)} \geq L_{\widehat{\boldsymbol{\theta}}_k(r)} \quad \forall r = 0, 1, 2, \ldots \tag{33}$$

with equality if and only if

$$\mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r+1), \widehat{\boldsymbol{\theta}}_k(r)\right) = \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right) \tag{34}$$

and

$$p_{\widehat{\boldsymbol{\theta}}_k(r+1)}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) = p_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{x}_k|\boldsymbol{y}_{1:k}) \tag{35}$$

for almost all (with respect to Lebesgue measure) $\boldsymbol{x}_k$.

Proof. See Appendix A. □

According to (3), the likelihood of the MP signal parameters in the M-step depends on the code delay of received signals via the cross correlation function. For the C/A code in the GPS L1 signal, the correlation function is a convex triangular function. Therefore, the strict monotonicity of any EM iteration is guaranteed in the context of MP mitigation in GNSS receivers.

In the proposed EM-based MP mitigation approach, $\mathcal{Q}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$ is approximated by using the particle-based approximation $\widehat{\mathcal{Q}}_{N_s}\left(\boldsymbol{\theta}_k, \widehat{\boldsymbol{\theta}}_k\right)$. According to Lemma 9.2 in [32], $\widehat{\mathcal{Q}}_{N_s}\left(\boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}\right)$ provides an accurate approximation of $\mathcal{Q}\left(\boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}\right)$ when the number of particle N_s is sufficiently large. Therefore, using a particle-based approximation $\widehat{\mathcal{Q}}_{N_s}\left(\boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}\right)$ instead of $\mathcal{Q}\left(\boldsymbol{\theta}, \widehat{\boldsymbol{\theta}}\right)$ is a reasonable way of implementing the ML estimation of MP signal parameters in the proposed EM-based MP mitigation approach.

5. Algorithm assessment

In order to evaluate the performance of the proposed EM-based MP mitigation approach, the GPS L1 C/A signal was taken into account and the related parameters used in all test scenarios are provided in Table 1. The MP mitigation approach was implemented for two kinds of test scenarios: (a) a static scenario where the joint estimate accuracy of the state vector (containing the direct signal parameters) and model parameter vector (containing the MP signal parameters) was evaluated under the condition of a set of specified model parameters (i.e., the MP signal parameter offsets are constant with respect to the direct LOS signal); (b) a dynamic scenario in which the joint estimate accuracy was evaluated under the condition of time-varying model parameters (i.e., the MP signal parameter offsets are time-varying with respect to the direct LOS signal). The bootstrap particle filter was implemented for recursively approximating the filtering pdf in the proposed EM-based MP mitigation approach and the number of particles was set to $N_s = 100$. Finally, the root mean square error (RMSE) of the estimator associated with the direct LOS signal was used as a performance measure. This measure is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N_m}\sum_{i=1}^{N_m}\left(\widehat{\boldsymbol{x}}^{(i)} - \boldsymbol{x}\right)^2} \tag{36}$$

where $\widehat{\boldsymbol{x}}^{(i)}$ is the ith run estimate. 100 Monte Carlo (MC) simulations ($N_m = 100$) were run for any test scenario. All algorithms have been coded using MATLAB and run on a laptop with Intel i-5 and 8 GB RAM.

5.1. MP mitigation performance comparison in static scenarios

The received complex baseband signals was generated by using the signal model in (1) and we assume that one reflected MP signal affects the direct LOS signal (i.e., $M = 1$). In order to evaluate the estimation performance of the proposed EM-based MP mitigation approach and to compare it with other MP mitigation approaches in the static scenario, we considered the proposed MP mitigation approach, the fast iterative maximum-likelihood algorithm (FIMLA) studied in [20] and the coherent narrow correlator DLL for this scenario. The maximum number of iterations in both the proposed MP mitigation approach and the FIMLA were set to 15 (i.e., $r_{\max} = 15$). The carrier-to-noise ratio (CNR) of the direct LOS signal was set to 50 dB-Hz and the multipath-to-direct ratio of the MP signal amplitude was maintained to a constant value 0.5. In addition, the RMSE envelops (i.e., the RMSE versus the code delay offset of MP signal) for the code delay and carrier phase of the direct LOS signal was used to evaluate the performance of the proposed MP mitigation method. A set of code delay offsets of the MP signal can be obtained by setting the difference between the code delays of the direct LOS signal and those of the MP signal in the interval $(0, 1.5T_c)$, i.e., $\Delta\tau_1 = \tau_1 - \tau_0$ where $\Delta\tau_1 \in (0, 1.5T_c)$. The carrier phase offset of the MP signal (i.e., $\Delta\varphi_1 = \varphi_1 - \varphi_0$) was set to 0° (in-phase with the direct LOS signal) and to 180° (out-of-phase with the direct LOS signal) when computing the RMSE envelops associated with the direct LOS code delay, and to 90° and -90° (orthogonal with the direct LOS signal) when computing the RMSE envelops for the direct LOS carrier phase, respectively.[1]

As shown in Fig. 1, the RMSE envelopes of the direct LOS code delay and carrier phase obtained with the proposed EM-based MP mitigation approach and the FIMLA are smaller than those obtained with the narrow correlator DLL, which indicates that the MP mitigation performance of the ML-based methods is better than that of the narrow correlator-based method. In addition, the estimation performance for the direct LOS carrier phase is similar for the proposed approach and FIMLA, whereas the RMSE envelop of the direct LOS code delay obtained with the proposed approach is obviously smaller than the other ones. Accordingly, estimated errors of the MP code delay (i.e., $\delta\tau_1 = \widehat{\tau}_1 - \tau_1$) obtained with the proposed approach and the FIMLA over 100 MC simulations are depicted as box plots in Fig. 2 (the figure only displays the estimated error for the in-phase situation since a similar result is

Table 1
Related parameters in test scenarios.

Amplitude noise	$\sigma_a = 0.01$
Code delay noise	$\sigma_\tau = 0.01$ chips/s
Doppler frequency noise	$\sigma_f = 5$ Hz/s
Integration time	$T_a = 25$ ms
Correlation spacing	$\delta = 0.15$ chips

[1] In a conventional tracking loop of the GPS receiver, the tracking errors of the direct LOS code delay due to MP interferences were maximized when the carrier phase offset of MP signal was 0° and 180° and the tracking errors of the direct LOS carrier phase due to MP interferences were maximized when the carrier phase offset of MP signal was -90° and 90°.

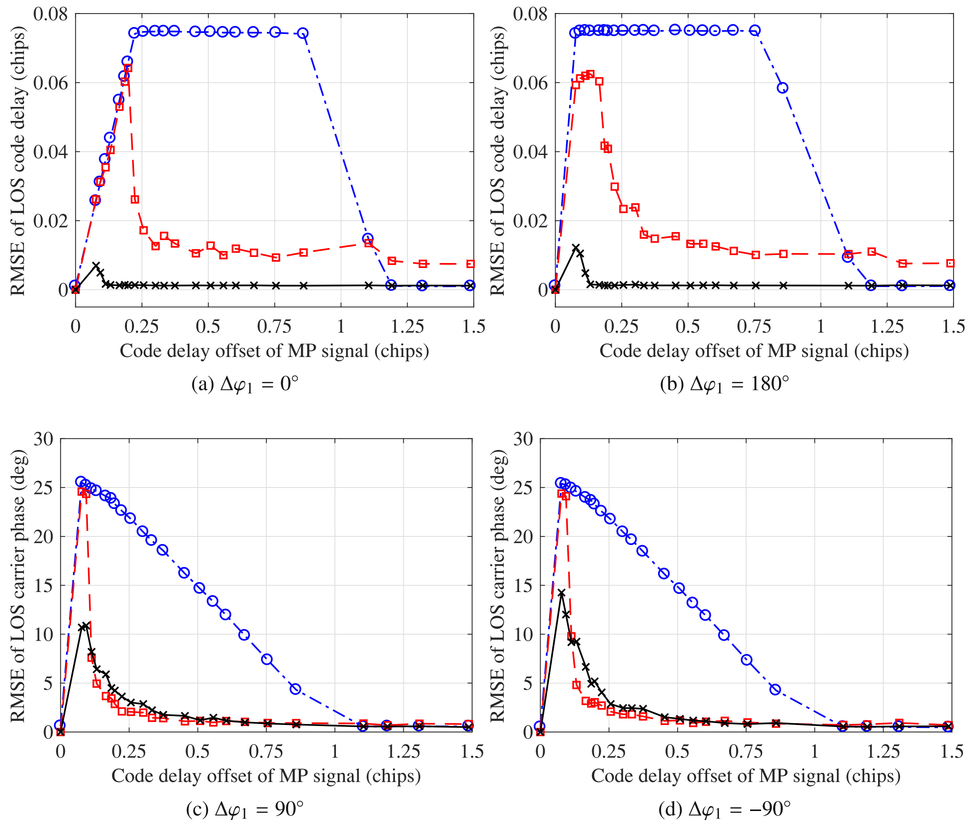


Fig. 1. RMSE envelopes of the LOS code delay and carrier phase with different approaches. Proposed approach: black solid line; FIMLA: red dashed line; Narrow correlator DLL: blue dash-dotted line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

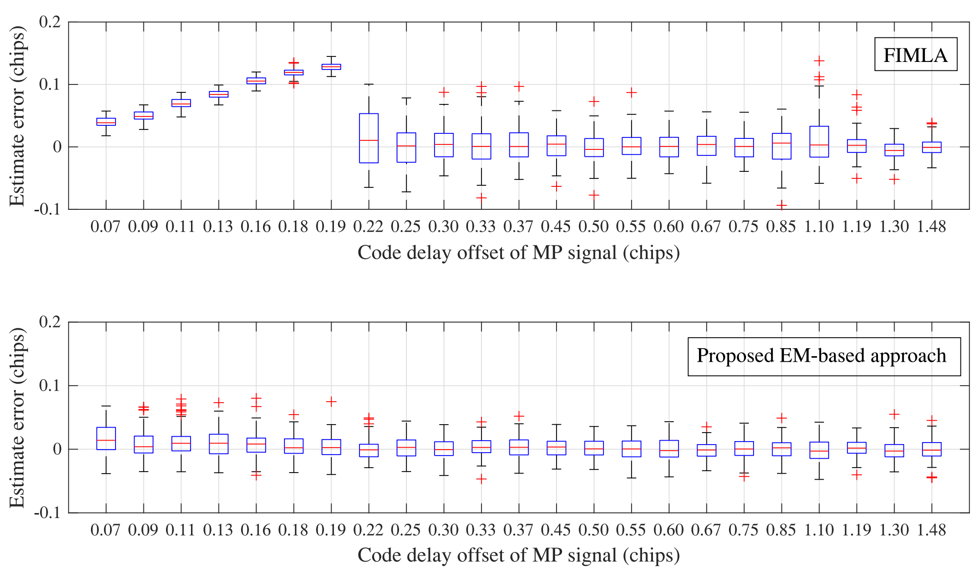
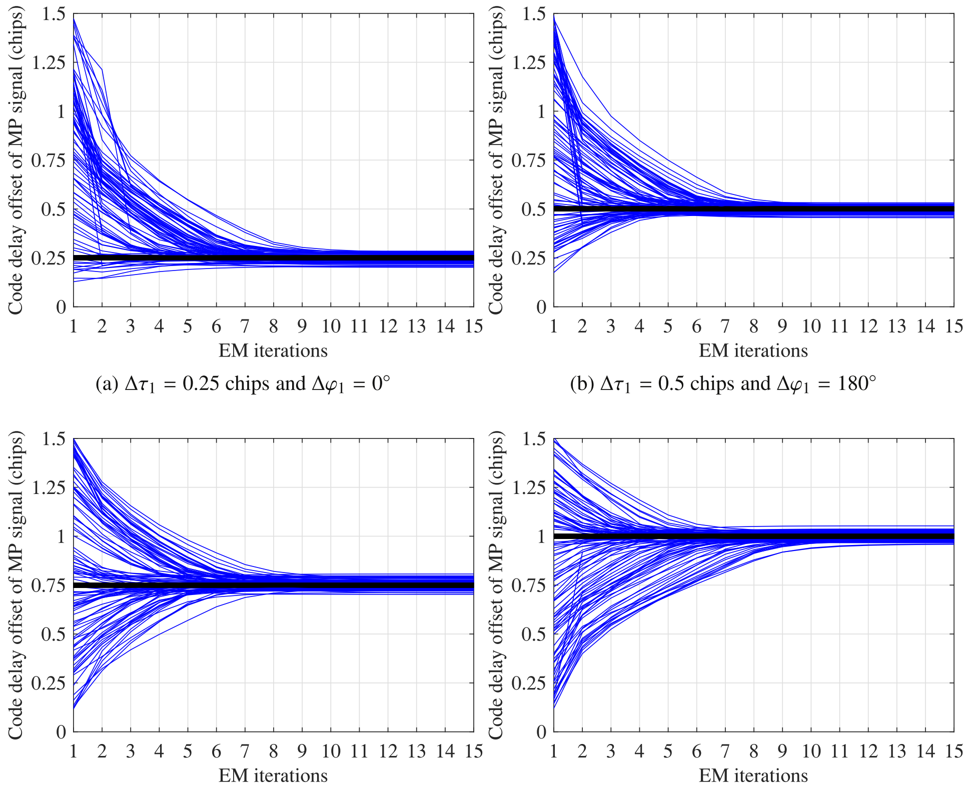


Fig. 2. Estimate error of MP code delay versus the code delay offset of MP signal.

obtained for the out-of-phase component). It is clear that the estimated error of the MP code delay obtained with the proposed approach is significantly smaller than that obtained from the FIMLA when the code delay offset of the MP signal is smaller than 0.2 chips. This difference between the two approaches gradually disappears as the code delay offset of MP signal increases. Although the direct LOS and MP signal parameters are jointly estimated based on the iterative fashion both in these two approach, the proposed approach can effectively improve the estimation performance due to the fact that the prior information about the direct LOS signal parameters is taken into account by using the EM iteration. Fig. 3 shows the estimated MP code delay offset (i.e., $\Delta\widehat{\tau}_1 = \widehat{\tau}_1 - \tau_0$) versus the EM iteration times over 100 MC simulations. Thanks to the convex property of the correlation function associated with the C/A code of the GPS L1 signal, the estimated MP code delay offsets converge rapidly to values close to the true parameters when the



(a) $\Delta\tau_1 = 0.25$ chips and $\Delta\varphi_1 = 0°$

(b) $\Delta\tau_1 = 0.5$ chips and $\Delta\varphi_1 = 180°$

(c) $\Delta\tau_1 = 0.75$ chips and $\Delta\varphi_1 = 90°$

(d) $\Delta\tau_1 = 1$ chips and $\Delta\varphi_1 = -90°$

Fig. 3. Estimated MP code delay offset versus the number of EM iterations for 100 MC simulations. Estimated value: blue line; True value: black line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

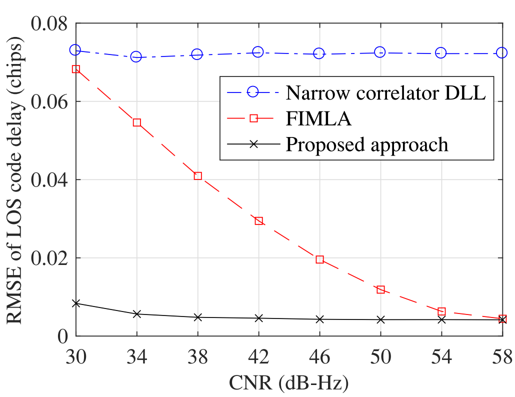


Fig. 4. RMSE of the LOS code delay for different CNRs.

corresponding initial value at the beginning of the EM iterations is uniformly chosen in the fixed interval.

In order to evaluate the effect of different CNR levels on the estimation performance of the proposed approach, the three MP mitigation approaches were implemented using code delay and carrier phase offsets of the MP signal fixed to 0.5 chips and 0°, respectively. Fig. 4 displays the RMSE of the direct LOS code delay versus CNR. It is known that the narrow correlator DLL is insensitive to CNR changes, which is confirmed in Fig. 4. Considering that a smaller value of CNR leads to a larger noise variance impairing the ML estimation accuracy, the FIMLA is very sensitive to the CNR and requires a long integration time (resulting in a small noise variance) to obtain reliable estimation results. The proposed approach using prior information about the direct LOS signal can efficiently reduce the influence of noise and MP on the estimation performance.

Considering that more than one MP signal enters the receiver at the same time in some MP scenarios, we assume in a second scenario that two reflected MP signals affect the direct LOS signal (i.e., $M = 2$). The multipath-to-direct ratios of the MP signal amplitudes were set to 0.7 and 0.4, respectively. The estimated mean and standard deviation of the code delay offsets for the two MP signals obtained with the proposed approach and FIMLA over 100 MC simulations are reported in Table 2. Table 2 shows that similar results are obtained when the two MP signals are in-phase and orthogonal. It is clear that the estimation performance of the approaches is degraded when the code delay offsets of the two MP signals are relatively close, whereas the corresponding estimation accuracy is clearly improved as the code delay offsets of the two MP signals are gradually separated. In addition, the estimated accuracy obtained with the proposed approach is better than that obtained with FIMLA, especially for the first MP signal (i.e., a short MP signal). Thus a better MP mitigation performance can be obtained by using the proposed EM-based MP mitigation approach in the presence of multiple MP signals.

5.2. MP mitigation performance in dynamic scenarios

In order to demonstrate the joint estimation performance of the proposed EM-based MP mitigation approach in dynamic scenarios, we considered one reflected MP signal with the following dynamics [23,25]:

(i) The code delay of the MP signal is assumed to follow the process

$$\begin{aligned} \tau_{1,k} &= \tau_{1,k-1} + \dot{\tau}_{1,k-1} T_a + \omega_\tau \\ \dot{\tau}_{1,k} &= \dot{\tau}_{1,k-1} + \omega_{\dot{\tau}} \end{aligned} \tag{37}$$

Table 2
Estimated means and standard deviations for the code delay offsets of the two MP signals.

True MP signal parameter offsets				Estimated MP code delay offsets			
code delay (chips)		carrier phase (°)		(mean ± standard deviation)			
$\Delta\tau_1$	$\Delta\tau_2$	$\Delta\varphi_1$	$\Delta\varphi_2$	Proposed approach		FIMLA	
				$\Delta\hat{\tau}_1$	$\Delta\hat{\tau}_2$	$\Delta\hat{\tau}_1$	$\Delta\hat{\tau}_2$
	0.18			0.025 ± 0.122	0.173 ± 0.043	0.058 ± 0.003	0.076 ± 0.004
	0.25			0.088 ± 0.057	0.208 ± 0.039	0.176 ± 0.059	0.207 ± 0.030
	0.30			0.080 ± 0.043	0.283 ± 0.056	0.027 ± 0.027	0.236 ± 0.032
	0.45			0.086 ± 0.026	0.434 ± 0.036	0.034 ± 0.008	0.387 ± 0.038
0.10	0.60	0°	0°	0.097 ± 0.019	0.595 ± 0.031	0.039 ± 0.006	0.554 ± 0.036
	0.82			0.096 ± 0.017	0.818 ± 0.026	0.041 ± 0.006	0.777 ± 0.036
	0.98			0.094 ± 0.026	0.970 ± 0.042	0.049 ± 0.013	0.949 ± 0.052
	1.19			0.097 ± 0.032	1.183 ± 0.069	0.046 ± 0.010	1.151 ± 0.043
	1.48			0.096 ± 0.012	1.479 ± 0.020	0.040 ± 0.006	1.478 ± 0.026
	0.18			0.124 ± 0.095	0.109 ± 0.059	0.001 ± 0.029	0.148 ± 0.064
	0.25			0.068 ± 0.049	0.239 ± 0.051	0.006 ± 0.031	0.172 ± 0.078
	0.30			0.076 ± 0.033	0.291 ± 0.045	0.023 ± 0.019	0.257 ± 0.033
	0.45			0.097 ± 0.017	0.451 ± 0.025	0.034 ± 0.008	0.439 ± 0.029
0.10	0.60	0°	90°	0.097 ± 0.015	0.604 ± 0.024	0.039 ± 0.007	0.593 ± 0.025
	0.82			0.095 ± 0.015	0.822 ± 0.027	0.042 ± 0.007	0.820 ± 0.026
	0.98			0.096 ± 0.014	0.979 ± 0.025	0.043 ± 0.007	0.979 ± 0.022
	1.19			0.099 ± 0.011	1.185 ± 0.023	0.049 ± 0.012	1.190 ± 0.018
	1.48			0.098 ± 0.012	1.482 ± 0.019	0.046 ± 0.006	1.478 ± 0.018

where $\dot{\tau}_{1,k}$ refers to the change rate of the MP code delay, ω_τ and $\omega_{\dot{\tau}}$ are zero mean Gaussian white noises with variances σ_τ^2 and $\sigma_{\dot{\tau}}^2$. In addition, the MP code delay is initialized with

$$\tau_{1,0} = \tau_{0,0} + |\Delta\tau_{1,0} + \omega_\tau| \quad (38)$$

where $\Delta\tau_{1,0}$ is the initial code delay offset of the MP signal with respect to the direct LOS signal.

(ii) The complex amplitude of the MP signal $\boldsymbol{\alpha}_{1,k}$ depends on the previous amplitude $\boldsymbol{\alpha}_{1,k-1}$ through the parametric model

$$\boldsymbol{\alpha}_{1,k} = e^{j2\pi f_{ca} T_a \dot{\tau}_{1,k}} \boldsymbol{\alpha}_{1,k-1} + \boldsymbol{\omega}_\alpha \quad (39)$$

where $\boldsymbol{\omega}_\alpha$ is a zero mean additive complex Gaussian white noise with variance σ_α^2.

This model was motivated by its efficiency in MP prone environments such as the urban canyons [27]. We propose to compare the proposed approach with FIMLA, the coherent narrow correlator DLL and particle filter-based MP mitigation approach studied in [22] for this scenario. Accordingly, the propagation models for the direct LOS and MP signal parameters used in the particle filter-based MP mitigation approach were defined using (5), (37) and (39). The simulation time T was set to 5 s. The CNR of the direct LOS signal and the multipath-to-direct ratio of the MP signal amplitudes were set to 46 dB-Hz and 0.5, respectively. A fast-fading MP condition was considered in this dynamic scenario. Accordingly, the initial code delay offset of the MP signal and the corresponding rate of change were set as $\Delta\tau_{1,0} = 0.2$ chips and $\dot{\tau}_{1,0} = 0.01$ chips/s (which is equivalent to a relative speed of about 4 m/s between the receiver and the reflector). The carrier Doppler frequency offset of the MP signal with respect to the direct LOS signal (i.e., the fading frequency) was approximately equal to 5 Hz. The standard deviations were fixed to $\sigma_\tau = 10^{-3}$ chips, $\sigma_{\dot{\tau}} = 10^{-4}$ chip/s and $\sigma_\alpha = 0.01$, which is a reasonable choice to resemble a typical urban satellite navigation channel environment [40].

Fig. 5 shows the RMSEs of the direct LOS code delay obtained with different approaches in the fast-fading MP condition. Since the prior information about the LOS signal parameters is considered both in the proposed EM-based and particle filter-based MP mitigation approaches, the RMSEs obtained with these two

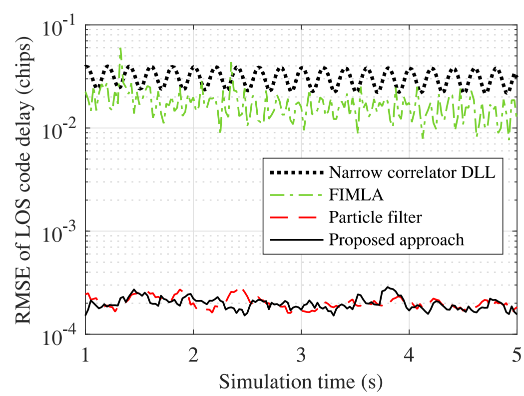


Fig. 5. RMSE of the LOS code delay obtained with the different approaches.

approaches are obviously smaller than those obtained with the coherent narrow correlator DLL and FIMLA. Moreover, the results obtained by using the proposed EM-based and particle filter-based MP mitigation approaches are very similar. Although the propagation model associated with MP signal parameters is not considered in the proposed approach, the EM iteration can reduce the impact of the model uncertainty resulting from the unknown model parameter vector (containing the MP signal parameters) on the estimate accuracy for the state vector (containing the direct signal parameters). The averaged values of MP code delay offset estimates over 100 MC simulations are depicted in Fig. 6. Since the EM iteration is implemented over each observation interval, the change of MP code delay with time can be accurately tracked by using the proposed approach and the corresponding estimation accuracy is slightly inferior to that obtained with the particle filter. This is due to the fact that the prior propagation model for MP signal parameters is not taken into account in the proposed approach. When the number of EM iterations is maximum, the computational complexities of the proposed EM-based and particle filter-based MP mitigation approaches are $O(r_{\max} N_s T)$ and $O(N_s T)$, respectively. Table 3 shows the execution times for 100 Monte Carlo runs by using different numbers of particles in the proposed EM-based and particle filter-based MP mitigation approaches. The computational cost of

Table 3
Execution times using the different number of particles.

Number of particles (N_s)	Execution times (s)		
	Proposed approach with the maximum number of iterations	Proposed approach with the stopping rule	Particle filter
50	191.34	105.16	36.58
100	351.85	192.20	69.29
200	662.38	360.47	138.91
400	1295.66	714.87	271.31

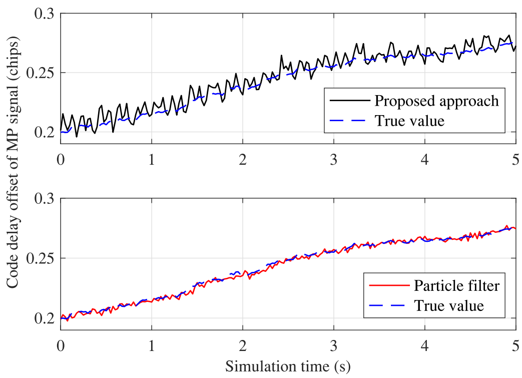


Fig. 6. Averaged values of MP code delay offset estimates over 100 MC simulations.

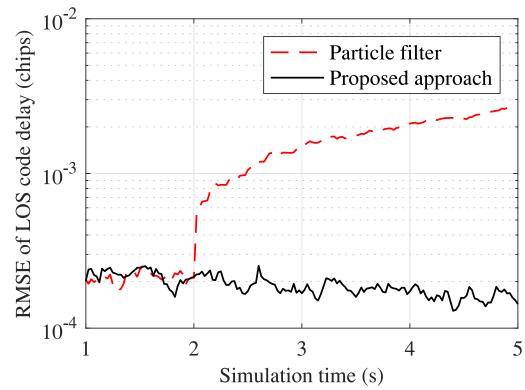


Fig. 7. RMSE of the estimated LOS code delay in the presence of changes affecting the dynamics of MP signals.

the proposed approach was evaluated in the following two situations: (a) the algorithm is stopped when the stopping rule in Line 8 of Algorithm 2 is satisfied (b) the number of EM iterations is set to its maximum value $r_{\max} = 15$. Since the implementation of the EM iteration is made for each observation interval, the computational cost of the proposed approach is much higher than the one associated with the particle filter. Thanks to the good convergence property of the proposed EM-based MP mitigation approach, the actual number of EM iterations of the proposed approach is less than the maximum number of iterations. As a consequence, the corresponding computational cost can be efficiently reduced by introducing the stopping rule of Line 8 of Algorithm 2 in the proposed approach.

According to the previous experiments, the particle filter-based approach has a better MP mitigation performance due to the fact that the propagation model for MP signal parameters follows the actual dynamics of MP signals. However, an accurate propagation model cannot always be defined to capture the dynamics of MP signals in real applications. In the next experiments, we assume that the differences between consecutive code delays of the MP signal are independent and follow an exponential distribution [41], i.e.,

$$\mu_{1,k} = \tau_{1,k} - \tau_{1,k-1} \tag{40}$$

with

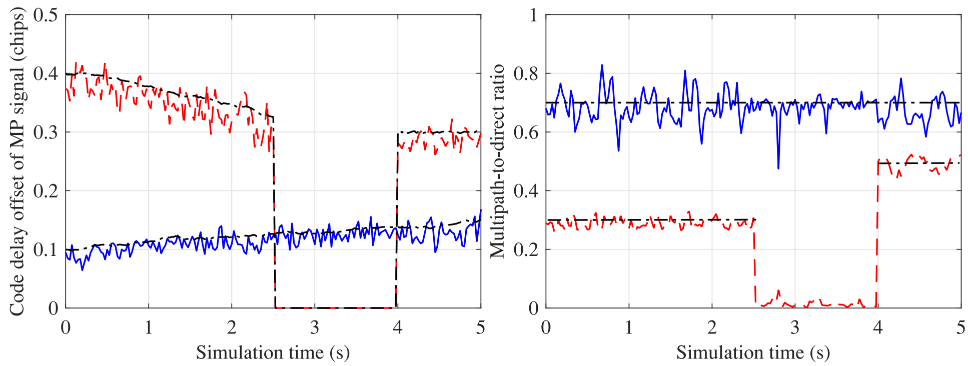
$$p(\mu_{1,k}) = \frac{1}{\mu_0} e^{-\frac{\mathbb{I}_{\mathbb{R}}(\mu_{1,k})}{\mu_0}} \tag{41}$$

where $\mathbb{I}_{\mathbb{R}^+}$ is the indicator function on $\mathbb{R}^+$ and μ_0 is a predefined decaying parameter, which was set to 2×10^{-3} chips in this experiment. In addition, the complex amplitude of the MP signal varies according to (39). In this experiment, the code delay of the MP signal was generated using (37) during the time interval [0 s, 2 s) and using (40) during the time interval [2 s, 5 s]. Fig. 7 displays the RMSEs of the direct LOS code delay obtained with the proposed EM and particle filter-based MP mitigation approaches. It is clear that the performance of the particle filter-based approach is significantly reduced during the second time interval [2 s, 5 s] due to an inaccurate model for the dynamics of MP signals. Conversely, the proposed EM-based approach is less affected by an inaccurate propagation model for the MP signal parameters, since this model is not used in the EM algorithm. These results indicate that the proposed EM algorithm provides a better robustness in constraint environments, such as urban canyons.

Table 4
MP signal parameters in dynamic scenarios.

Signal parameters	MP signals		
	MP #1	MP #2	
Appearance time (s)	0–5	0–2.5	4–5
Initial code delay offset (chips)	0.1	0.4	0.3
Initial change rate of the MP code delay (chips/s)	0.01	−0.03	0
Multipath-to-direct	0.7	0.3	0.5
Fading frequency (Hz)	2	5	0

In the last experiments, we consider two reflected MP signals simultanously affecting the direct LOS signal dynamically. The parameters of the two MP signals are provided in Table 4. The estimated code delay offsets and multipath-to-direct ratios of these MP signals are displayed in Fig. 8. As shown in Fig. 8, the MP signal parameters can be accurately tracked by the proposed approach even in the presence of multiple MP signals. Note that the estimated value of the multipath-to-direct ratio for the MP signal #2 is less than 0.05 chips when the corresponding MP signal disappears. Thus a threshold on the estimated MP signal amplitude can be used for eliminating the false MP signal path estimation in the proposed MP mitigation approach. In other words, the MP signal path is not taken into account when the estimated MP signal amplitude is smaller than the threshold. Considering that two reflected signals very close in time can be considered as only one perturbation in the LOS signal parameter estimation [19], a two-path model (i.e., $M = 2$) in the proposed EM-based MP mitigation approach is often sufficient for practical applications.



(a) Estimated MP code delay offsets (b) Estimated multipath-to-direct ratios

Fig. 8. Estimation results for two MP signal parameters in dynamic scenarios. MP #1: blue solid line; MP #2: red dashed line; True value: black dash-dotted line. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

6. Conclusion

This paper proposed to mitigate the presence of MP interferences potentially affecting GNSS signals by jointly estimating a state vector containing the parameters of the direct signal and time-varying parameters associated with reflected signals. The estimation of all these parameters was conducted using an EM algorithm, whose convergence properties have been established in this context. Since the MP signal parameters were assumed to be time-varying with an unknown sufficient statistics, we studied an EM algorithm defined for each observation interval of the receiver for the joint estimation of the direct LOS and MP signal parameters. A simulation study was conducted in static and dynamic scenarios in order to compare the performance of the proposed approach with the narrow correlator delay lock loop (DLL), the algorithm FIMLA of [20] and the particle filter-based MP mitigation approach investigated in [22]. The proposed algorithm exploited some prior information about the direct LOS signal parameters via the different EM iterations, providing better MP interference mitigation when compared to the other strategies, especially for short MP signals. Although the estimation accuracy of the proposed approach was slightly overcome by the particle filter-based MP mitigation approach, the proposed EM algorithm showed a better robustness in MP environments since it does not require an accurate propagation model for the dynamics of MP signals. We think that the proposed MP mitigation technique is interesting for high chip rate GNSS signals, especially when these signals are contaminated by short delay MP interferences. In order to further reduce the computational load, our future work will be devoted to implementing a low-cost solution (such as unscented Kalman filter) for non-linear systems in the proposed EM algorithm. Testing the proposed algorithm in more practical applications using aircraft or autonomous car data is also an interesting prospect.

Appendix A. Proof of Corollary 1

Proof. According to Theorem 2 in [39], we have

$$L_{\widehat{\boldsymbol{\theta}}_{1:k}(r+1)}(\boldsymbol{y}_{1:k}) - L_{\widehat{\boldsymbol{\theta}}_{1:k}(r)}(\boldsymbol{y}_{1:k}) \geq \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_{1:k}(r+1), \widehat{\boldsymbol{\theta}}_{1:k}(r)\right) - \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_{1:k}(r), \widehat{\boldsymbol{\theta}}_{1:k}(r)\right) \geq 0. \tag{A.1}$$

Using (14) into (A.1) leads to

$$\sum_k L_{\widehat{\boldsymbol{\theta}}_k(r+1)}(\boldsymbol{y}_k) - \sum_k L_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{y}_k) \propto \sum_k \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r+1), \widehat{\boldsymbol{\theta}}_k(r)\right) - \sum_k \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right) \geq 0. \tag{A.2}$$

Accordingly, (A.2) can be decomposed as follows

$$L_{\widehat{\boldsymbol{\theta}}_k(r+1)}(\boldsymbol{y}_k) - L_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{y}_k) \propto \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r+1), \widehat{\boldsymbol{\theta}}_k(r)\right) - \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right) \geq 0 \tag{A.3}$$

where $k = 1, \ldots, \infty$. Considering that $L_{\widehat{\boldsymbol{\theta}}_k(r+1)}(\boldsymbol{y}_k) - L_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{y}_k)$ is an increasing function of $\mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r+1), \widehat{\boldsymbol{\theta}}_k(r)\right) - \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right)$ for the kth observation interval, $L_{\widehat{\boldsymbol{\theta}}_k(r+1)}(\boldsymbol{y}_k) \geq L_{\widehat{\boldsymbol{\theta}}_k(r)}(\boldsymbol{y}_k)$ holds when $\mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r+1), \widehat{\boldsymbol{\theta}}_k(r)\right) \geq \mathcal{Q}\left(\widehat{\boldsymbol{\theta}}_k(r), \widehat{\boldsymbol{\theta}}_k(r)\right)$. □

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