Robust Kalman Filtering for NLOS Mitigation of GNSS Measurements in Urban Environments

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ABSTRACT- It is well-known that the Extended Kalman Filer (EKF) is the standard estimation method for positioning with GNSS measurements. However, this filtering method is not optimal when the GNSS measurements become contaminated by non-Gaussian errors including multipath (MP) and non-line-of-sight (NLOS) errors. In this paper, we apply some techniques from robust statistic to make the conventional EKF more resistant to outliers which may be summed up as MP and NLOS signals in urban environments. We study two robust estimators that do not require tuning parameters fixed in advance: the first estimator detect the outliers using a robust statistical test based on the measure of the distance between each innovation sample with respect to the median of all innovations, then assign to them a low weight in the state estimation while keeping nominal weights for good Pseudo-Ranges (PR). The second estimator exploits the difference between two successive innovations to detect jumps related to large errors as MP and NLOS bias and correct their effect via a new recursive weighting technique. Test results using real GPS signal in downtown of Toulouse show that these estimators are simple to implement and capable of detecting multiple outliers in real-time and then improving the positioning accuracy compared to the conventional EKF.

INTRODUCTION

Although there is an exponential increase of global navigation satellites system (GNSS) applications in urban environments, these services are looking forward for mature and robust geolocation solutions for urban and indoor settings. The main reason of this gap between user expectations & requirements from one side and the existing technologies from other side is that these environments present significant challenges for satellites positioning. The high density of tall buildings and the presence of many obstacles blocking signals during their propagation pose very challenging technical issues for acquiring and tracking the degraded GNSS signals. Building and other objects surrounding the receiving antenna may block the direct line-of-sight (LOS) signal from many satellites, hence reducing the visibility [1]. In addition, the interaction with the environment usually results in a superposition of various signals that have followed different paths. This situation produces a distortion of the pseudo-range (PR) measurements. Then, the GNSS receiver delivers biased position estimation in this kind of harsh environments.

In order to improve the performance of satellite navigation in urban environments, many of existing techniques aim to model these degradations and mitigate their effects at the level of signal processing, measurements or position domain [2]. In urban environments, the multipath and the non-line-of-sight (NLOS) signals will led to PR measurements populated with outliers in the navigation stage. Hence, it is essential to detect the degraded measurements to either delete them or decrease their impact on the user state estimation. However, these constrained environments induce generally a poor geometrical constellation and less received signals which led to a problem of redundancy with less than four signals available in some situations. Then, to handle this lack of information in these environments, one common approach is to use all the available signals while paying constant attention to reduce the effect of biased PR measurements. This is the principle of the robust Extended Kalman Filter (EKF) [3, 7]. We adopt this approach in this work for it simplicity with the objective to develop a practical and efficient implementation.

Since the conventional least squares method is not robust to outliers [4], Huber [5] introduced a new class of estimators, called M-estimators, which attempt to minimize other residual functionals instead of the sum of squares of residuals. These approaches substitute the conventional weighted least square solution which solves the equation (1):

$$x^{+} = \underset{x}{\operatorname{argmin}} \ \left\| y - H \cdot x \right\|_{W} = \underset{x}{\operatorname{argmin}} \left[\left(y - H \cdot x \right) \cdot W \cdot \left(y - H \cdot x \right)^{T} \right] \tag{1}$$

by a more robust solution minimizing a new convex cost function $\rho(.)$, called the influence function [3], and solving this equation (2):

$$x^{+} = \underset{x}{\operatorname{argmin}} \ \rho(y - H \cdot x) . \qquad (2)$$

This yields solutions that verify the following equality (3) [3]:

$$H \cdot D(e) \cdot e = H \cdot D(y - H \cdot x) \cdot (y - H \cdot x) = 0 \ , \qquad (3)$$

where $D(e) = \mathrm{diag}(\rho^{'}(e_1)/\mathrm{e}_1 , \rho^{'}(e_2)/\mathrm{e}_2 , \ldots, \rho^{'}(e_N)/\mathrm{e}_\mathrm{N})$ is the weighting matrix, $e = (e_1, \ldots, e_N)$ is the residual error and N is the number of measurements. By contrast, the conventional least squares method yield solutions that verify this usual relation (4) leading to the weighted Least Squares solution (WLS):

$$H \cdot W \cdot e = H \cdot W \cdot (y - H \cdot x) = 0 . \qquad (4)$$

By applying a weight depending on the residual error instead of equal nominal weights, M-estimators ensure the robustness of the estimation: assigning a weight to each Pseudo-Range (PR) measurement to suppress the impact of possible outliers and define its contribution to the state estimate at each time epoch. Reference [3] uses this robustness ability that M-estimators have to propose a new robust EKF by weighting the innovation vector at each time step. Reference [6] changes the minimization problem (2) by introducing a new term leading to a new re-weighted EKF. This estimator applies the weight function of an M-estimator to a transformed innovation $I_k = R_k^{-\frac{1}{2}} \cdot \left(y_k - H_k \cdot x_k\right)$ and introduces a new weighted measurement covariance matrix $R_W = (R_k^{-\frac{1}{2}} \cdot W_\mathrm{k} \cdot R_k^{-\frac{1}{2}})^{-1}$ with R_k is the measurement covariance matrix at time step k. Other versions of robust EKF have been published in the literature. Different approaches use weight matrix obtained from different influence functions and weighting strategies. But each influence function gives solutions with different robustness properties which make the choice of the appropriate function for each application a challenging issue. Reference [3] proposes a three parts influence function, while [5] suggests a two parts influence function splitting the residuals errors into two groups according to it standard deviation. Reference [4] introduces a new L_1-L_2 weighting function that combine the benefits of reducing the outliers influence using the L_1-norm and obtaining convex objective function using the L_2-norm. Reference [6] relies on Hample's three parts redescending M-estimator to present a new modified and damped influence function and [7] argues for a modified Rao influence function based on the analysis of measure histograms. The choice of the appropriate function is based on a priori statistical analysis of the environment but the need of tuning threshold parameters for optimal performance makes the M-estimators difficult to use in real-time applications. Besides, the risk of ending up with deteriorated performances renders these estimators inappropriate to high-variable environments with multiple sources of outliers.

In this paper, we are interested in circumventing the problem of adaptability of M-estimators parameters by investigating in pre-processing approaches to compute the weight matrix at each time epoch for more robust and autonomous state estimation in real-time applications. In the second section, we propose a Robust Extended Kalman Filter (REKF) that use statistical test on residuals to detect outliers (i.e. degraded PR measurements), compute the weights to down-weight outliers at each time epoch adaptively to the input PR measurements. In the third section, we make use of the difference between the residual at the current time step and the previous one to build a recursive weighting strategy based on the detection of excess change of innovation. In section 4, we show the results of robustness properties that our methods provide using real GPS C/A measurements. Finally, some conclusions are summarized in section 5.

ROBUST EXTENDED KALMAN FILTER (REKF)

We aim to find a new simple adaptive weighting approach to make EKF more robust to outliers. Since the outliers present in PR measurements are hard to identify, we will focus on the innovation information to detect outliers. In each time epoch, we apply statistical test on the measurement errors to identify the outliers in the measurement. This approach is easy to implement to gain improvement in some urban environments with enough satellite visibility but with degraded measurements. If only one PR measurement from a satellite is contaminated by outliers, theses errors will appear at the innovation level making the corresponding measurement errors from this satellite more important than the measurement errors from non-contaminated satellites (which are almost at the same level of noise). Once this outlier is detected, the weight of the corresponding PR measurement will be set to the inverse of the distance between this outlier value and the median value of the innovation vector. We ensure then that this PR measurement will have a small contribution on the state estimate. The weight of inlier points (i.e. PRs which are not detected by the statistical test) will be set to 1 which means a total contribution to the PVT calculation. This new strategy is summarized in Fig. 1:

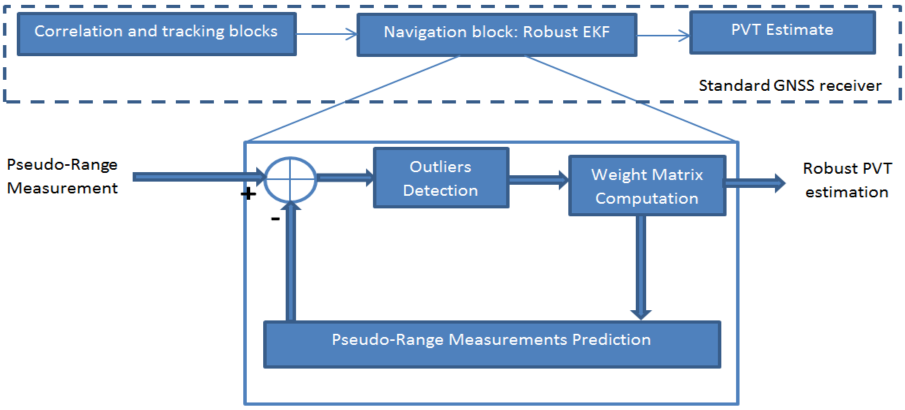

Fig. 1. High-level block diagram of the Adaptive Robust Extended Kalman Filter estimator

After linearization of the measurements equation this equation is expressed as:

$$y_k = H_k \cdot x_k + b_k + n_k \tag{5}$$

where: y_k is the PR measurements vector at time step k [N], x_k is the state vector at time step k [M], H_k is the observation matrix at time step k [N, M], b_k is the bias at time step k [N] and n_k is the zero-mean Gaussian white noise characterized by a covariance matrix R_k. If an outlier caused by Multipath or NLOS bias occurs on a PR measurement from a satellite, the innovation error distribution will change. The presence of Multipath error in the measurements yields a variance jump in the innovations [8] while the presence of NLOS errors introduces a mean jump in these residual measurements [8]. In the nominal case when all PR measurements are uncontaminated with outliers, the elements of observed residual errors at time step k are confined into a region of noise that depends on the maximum noise variance. Fig. 2 shows an example of innovation probability density (pdf) with contaminated and uncontaminated PR measurements:

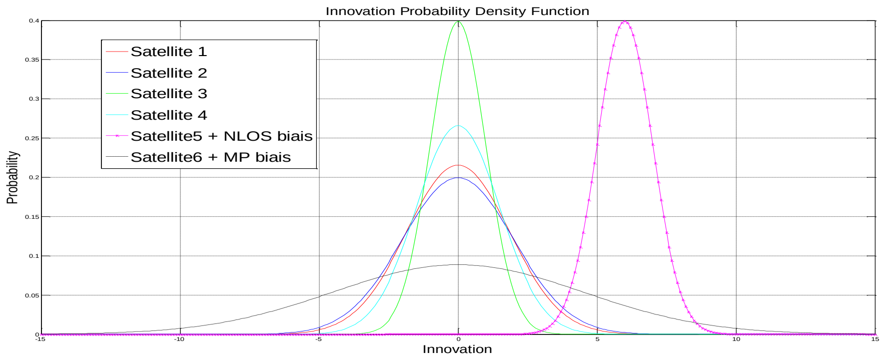

Fig. 2. Residual error distribution from N different satellites

Accordingly, the additive outlier in a PR measurement from a satellite j will induce a non-zero mean bias $\left((b_k)_j + (n_k)_j\right)$ that comes out the usual region of the noise at time step k. Using statistical tests on the residuals vector at time step k such as Chauvenet's criterion [9], the outlier could be detected since the corresponding residual will be far away from the median of the standard residuals. Chauvenet's criterion permit to determine whether a value is abnormal compared to the other signal values. The idea behind is to find a probability band containing all the normal samples of a data set. A suspected sample that has a value too far from the median of the data samples i.e. more than the median absolute deviation is considered as an outlier. Once a contaminated PR measurement $(y_{\mathrm{k}})_j$ is identified, the corresponding weight will be down to:

$$(w_k)_j = \min\left[1,\ 1\Big/\left|(y_{\mathrm{k}} - H_{\mathrm{k}}\cdot x_k)_j - median(y_{\mathrm{k}} - H_{\mathrm{k}}\cdot x_k)\right|\right]. \tag{6}$$

The idea is to ensure a positive weight not exceeding 1. This weight will be smaller if the residual value from the satellite j deviates more and more from the residual-vector median value. We choose the median value instead of the mean value because it is a robust measure of central tendency. This deweighting technique allows assigning a low weight to outlier measurements while giving the normal measurements a total contribution to the PVT computation. We compute then the state estimation at step time k using the obtained weighting matrix $W_{\mathrm{k}} = diag((w_k)_j)$. We introduce this matrix to weight the measurement covariance matrix as it is done in [6], $R_W = (R_k^{-\frac{1}{2}}\cdot W_{\mathrm{k}}\cdot R_k^{-\frac{1}{2}})^{-1}$. The main difference between the proposed method and the approach of [6] is that we compute the weighting matrix W_{k} using a statistical test based on the innovations median as defined in equation (6), while in [6] W_{k} is computed by applying an influence function of an M-estimator on a transformed innovation vector.

RECURSIVE ROBUST EXTENDED KALMAN FILTER (RREKF)

Sometimes, the transition between LOS and NLOS is sudden and can be detected due to the large errors associated to the NLOS. To exploit this fact, we study techniques as reference [10] which suggests a recursive weighting to make the conventional least square more robust to outliers. We analyze the recursive relation between weights applied to each PR measurement by comparing the innovation (i.e. difference between measured PR and predicted one) at two successive time steps. The linearized measurement vectors at time step (k-1) and k are defined as:

$$y_{\mathrm{k-1}} = H_{\mathrm{k-1}}\cdot x_{k-1} + b_{k-1} + n_{k-1} \tag{8}$$

$$y_{\mathrm{k}} = H_{\mathrm{k}}\cdot x_k + b_k + n_k \tag{9}$$

We suppose that we have well-weighted the measurements at time step (k-1) and we want to find the optimal weights at the time step k by detecting the excess change of the innovation between two successive time epochs [12]:

$$(\Delta \mathrm{e})_{\mathrm{j}} = (e_{\mathrm{k}} - e_{k-1})_j = (b_k - b_{k-1})_j + (n_k - n_{k-1})_j \tag{10}$$

When the measurement noise from each satellite is a zero-mean white Gaussian noise, the second term in the above difference lies between $-6\times\sigma_{\mathrm{j}}$ and $6\times\sigma_{\mathrm{j}}$ with a probability of 99% with σ_{j} is the measurement noise variance since each measurement error is within the interval $\lfloor -3\times\sigma_{\mathrm{j}}, 3\times\sigma_{\mathrm{j}}\rfloor$ at 99%. In the presence of outliers in the epoch time k and k-1, the innovation difference $(\Delta \mathrm{e})_{\mathrm{j}}$ should be within this interval. If the measurement from the satellite j is contaminated with outliers and the difference between the outliers bias is small i.e. $(b_k - b_{k-1})_j \approx 0$, then the innovation difference $(\Delta \mathrm{e})_{\mathrm{j}}$ should lie within the 4-sigma noise interval. In both cases, the weight at time step k, corresponding to the PR measurement from the satellite j should be set equal to the weight at time step (k-1), $(w_{k-1})_j$, meaning that we keep the same weight because the measurements are well-weighted at time step (k-1). If the innovation difference is outside the interval $\lfloor -6\times\sigma_{\mathrm{j}}, 6\times\sigma_{\mathrm{j}}\rfloor$, then we have to change the weight $(w_k)_j$. In this specious case, we assign a lower weight to the corresponding PR measurement equal to:

$$(w_k)_j = \min\left[1,\ 1\Big/\left|(\Delta e_k)_j\right|\right] \tag{11}$$

The idea is to ensure a positive weight not exceeding 1. This weight will get smaller if the outlier from the satellite j at time step k b_k is increasing so that the innovation difference is getting bigger. To summarize, the weight corresponding to the PR measurement from the satellite j will be equal to:

$$(w_k)_j = \begin{cases} (w_{k-1})_j & if \quad |(\Delta e_k)_j| \leq (6 \times \sigma_j) \\ \min\left[1, \ 1/|(\Delta e_k)_j|\right] & if \quad |(\Delta e_k)_j| > (6 \times \sigma_j) \end{cases} \tag{12}$$

Practically speaking, the measurement noise variance σ_j is unknown. So, we define the threshold $(6\times\sigma_j)$ as the maximum MP/NLOS admissible bias. We compute then the state estimation at time step k using the obtained weighting matrix $W_k = diag((w_k)_j)$. We introduce this matrix to weight the measurement covariance matrix as in [6]. To initialize the first weights, we use the weights obtained by the REKF estimator presented in the previous section.

Similarly to the REKF, this new strategy allows a new simple way of weighting the PR measurements without parameters tuning in advance as in M-estimation based robust Kalman filters.

RESULTS AND ANALYSIS

To assess the performance of the proposed positioning algorithms of GNSS, preliminary results are obtained from measurements collected in Toulouse downtown. The recorded PR measurements contain 14833 epochs corresponding to 58 minutes of recorded data. The data for the experiments were collected along the trajectory displayed in figure (3) of downtown Toulouse that includes narrow streets and high buildings which reduced the visibility of the satellites.

Fig. 3. Trajectory for data collection in downtown Toulouse [12]

We used the UBLOX 6T receiver and a SPAN Novatel system including a DGPS receiver tightly integrated with an IMU-FSAS (from iMAR). We consider the trajectory provided by the Novatel receiver as the reference trajectory for comparison with our algorithms. We compare the PVT solutions from the REKF, the RREKF, the conventional EKF and the robust EKF in [6] with a damped-Hampel M-estimator. We show in the following figure 4 the results of post-processing the GPS L1 C/A code PR measurements these fours methods.

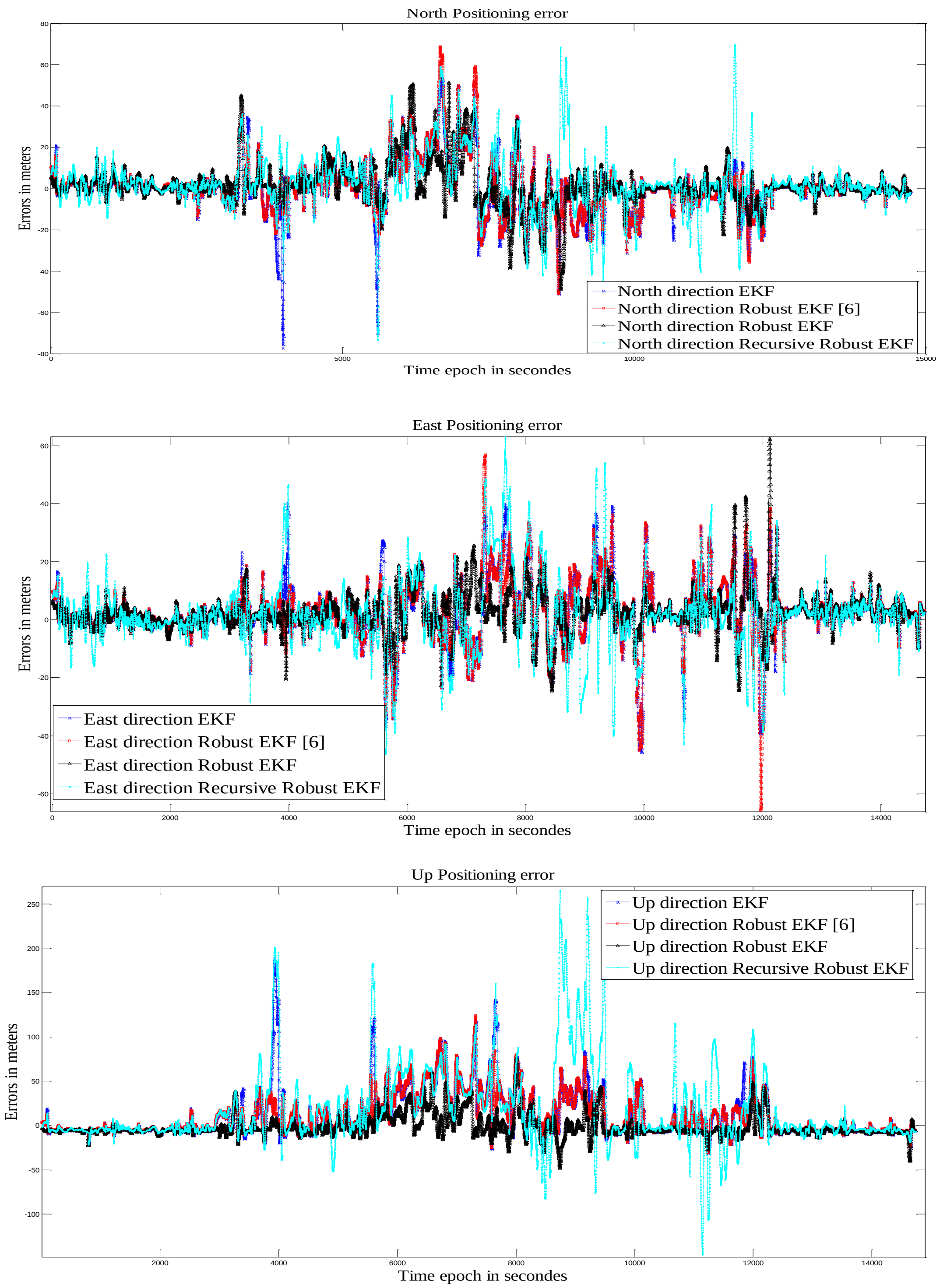

Fig. 4. North Positioning error (top), East Positioning error, and Up Positioning error (bottom)

The positioning error variation shows that REKF is more robust to MP and NLOS errors and produces less positioning error than the other estimators. The outliers in PR measurements caused by the deep urban environment induce big degradations on the positioning that the conventional EKF cannot handle. This led to a bad positioning accuracy in the three ENU directions. The robust estimators introduced in this paper can detect in real time these outliers and underweight their effect leading to more precise position in the three ENU directions. To highlight this result, we compute the median, the 5th percentile, the 95th percentile and the maximum value of the positioning error in each ENU direction and for all estimators:

Table 1.Performance evaluation of the four EKF estimators

	North Direction				East Direction			
	Median Positioning error [m]	5^{th} percentile [m]	95^{th} percentile [m]	Maximum Positioning error [m]	median Positioning error [m]	5^{th} percentile [m]	95^{th} percentile [m]	Maximum Positioning error [m]
Conventional EKF	0.4692	-20.9646	23.9384	77.2746	2.1222	-12.7793	21.3046	45.7713
Robust EKF: Perälä M-estimator	0.5069	-18.2333	23.5348	68.9710	2.0394	-12.1223	19.2631	65.8632
Robust EKF	0.5897	-12.7392	20.1790	51.4920	2.1260	-7.0352	13.7048	64.7134
Recursive Robust EKF	1.3597	-17.6996	28.0573	73.4195	1.4294	-17.6984	25.6776	63.0848

	Up direction			
	Median Positioning error [m]	5^{th} percentile [m]	95^{th} percentile [m]	Maximum Positioning error [m]
Conventional EKF	-1.1430	-9.7344	61.7679	187.6790
Robust EKF: Perälä M-estimator	-1.1049	-9.4923	55.9389	123.8774
Robust EKF	-5.2378	-13.2703	24.5521	76.8256
Recursive Robust EKF	-0.2618	-21.7245	106.1452	265.3133

It can be seen that the REKF improves the positioning accuracy while adapting the estimation in real-time to the outliers introduced into the PR measurements. This estimator is also simple to implement. It gives good positioning results with just simple modification of the conventional EKF. Regarding the RREKF estimator, it improves a little bit the positioning accuracy but it doesn't give the expected performances. Despite this performance-improvement, the use of REKF, RREKF estimators and the robust estimator in [6] lead to erroneous results when the outlier measurements have almost the same value (almost the same MP/NLOS bias for many PR measurements) or when the inlier measurements are less than 4 which explain the enormous positioning error in some deep urban environment as shown in the following figure (5) with PR measurements corrupted by MP from buildings and trees:

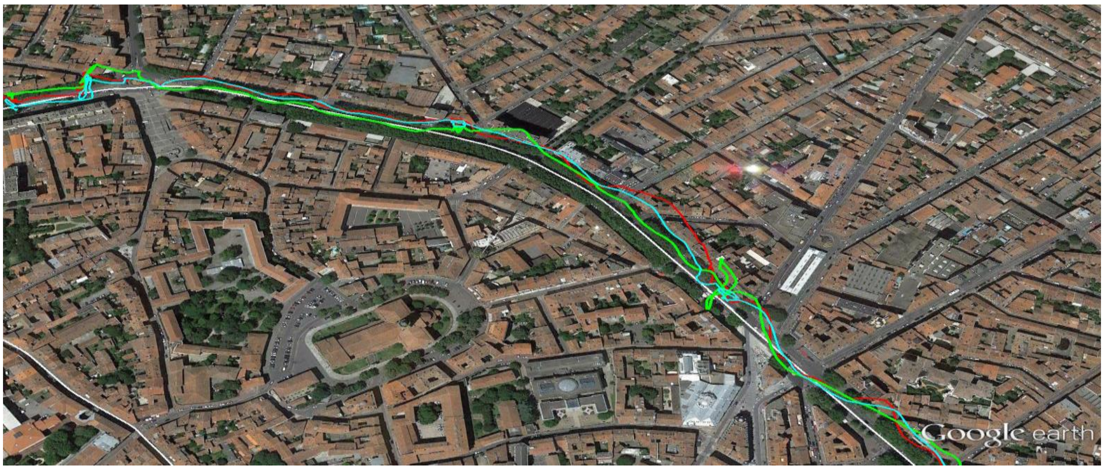

Fig. 5. Estimators and reference trajectories [12] with: white color refers to the reference trajectory, green to the REKF trajectory, red to the robust EKF in [6] trajectory and light blue to the RREKF trajectory.

CONCLUSIONS AND FUTUR WORKS

The purpose of this study is to implement simple techniques for rendering the conventional EKF more robust to outliers which may occur in urban environments without the need of tuning parameters fixed manually in advance as in the M-estimation. In this work, two robust estimators were presented: one adaptive robust estimator capable of real-time outliers-detection in the innovation and underweighting these outlier measurements, and another recursive robust estimator able to use the innovation difference between two time steps to build up a recursive weighting technique. Based on the obtained results, the two proposed methods seem to outperform the conventional EKF when outlier measurements occur. The REKF estimator gives better positioning performances than the RREKF estimator that does not give the expected improvements and has to be re-worked. Then, robust estimation methods should be used with caution for GNSS positioning in urban environments. But generally speaking, these estimators provide an autonomous robust EKF for MP/NLOS mitigation purposes in urban environments with enough redundancy of satellites in visibility which is suitable in a multi-constellation context.

This work confirm our prior conclusion that in deep urban canyons, even with advanced and robust signal processing algorithms, additional sources of positioning information are needed including 2D and 3D maps [13], navigation sensors and communication assistance.

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