Managing Noisy and Missing Measurements in Time Scale Generation for a Swarm of Nanosatellites

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1. Space interferometry

Radio interferometry measurements rely on stable timing systems to obtain consistent Times of Arrival (TOA).

Placing the telescope array in lunar orbit allows it to be shielded from Earth-based radio interference [1].

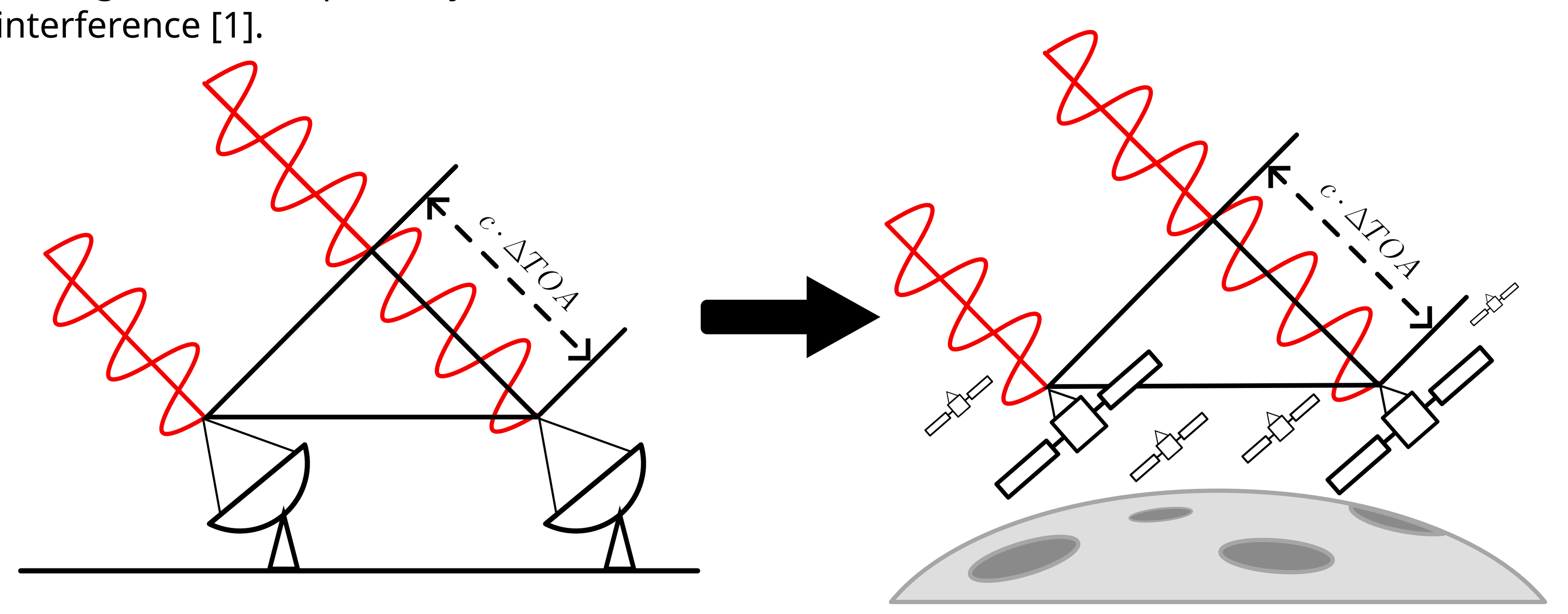


Figure 1: Radio interferometry depends on precision of time of arrival (TOA) measurements, whether based on land or in space.

2. Time Scales

The Basic Time Scale Equation details how satellite "i" can realize it's offset from the time scale "E" [2]:

$$x_{i,E}(t) = h_i(t) - h_E(t) = \sum_{j=1}^{N} w_j(\hat{x}_{j,E}(t) - z_{j,i}(t)) \quad (1)$$

The key components of the BTSE are:

- Clock phase predictions: $\hat{x}_{i,E}(t) = x_{i,E}(t) + \tau y_{i,E}(t-\tau) + \frac{\tau^2}{2} d_{i,E}(t-\tau) \quad (2)$
- Clock bias measurements: $z_{j,i}(t) = h_j(t) - h_i(t) + n_{j,i}(t) \quad (3)$
- Weights: $w_j(t) = f\left(\hat{x}_{j,E}(t) - z_{j,i}(t)\right) \quad (4)$

3. Anomalies

Clock phase jumps:

$$\tilde{h}_i(t) = h_i(t) + \Delta h \quad (5)$$

Clock frequency jumps:

$$\tilde{h}_i(t) = h_i(t) + \Delta y(t - t_a) \quad (6)$$

Measurement anomalies:

$$\tilde{z}_{j,i}(t) = h_j(t) - h_i(t) + \tilde{n}_{j,i}(t) \quad (7)$$

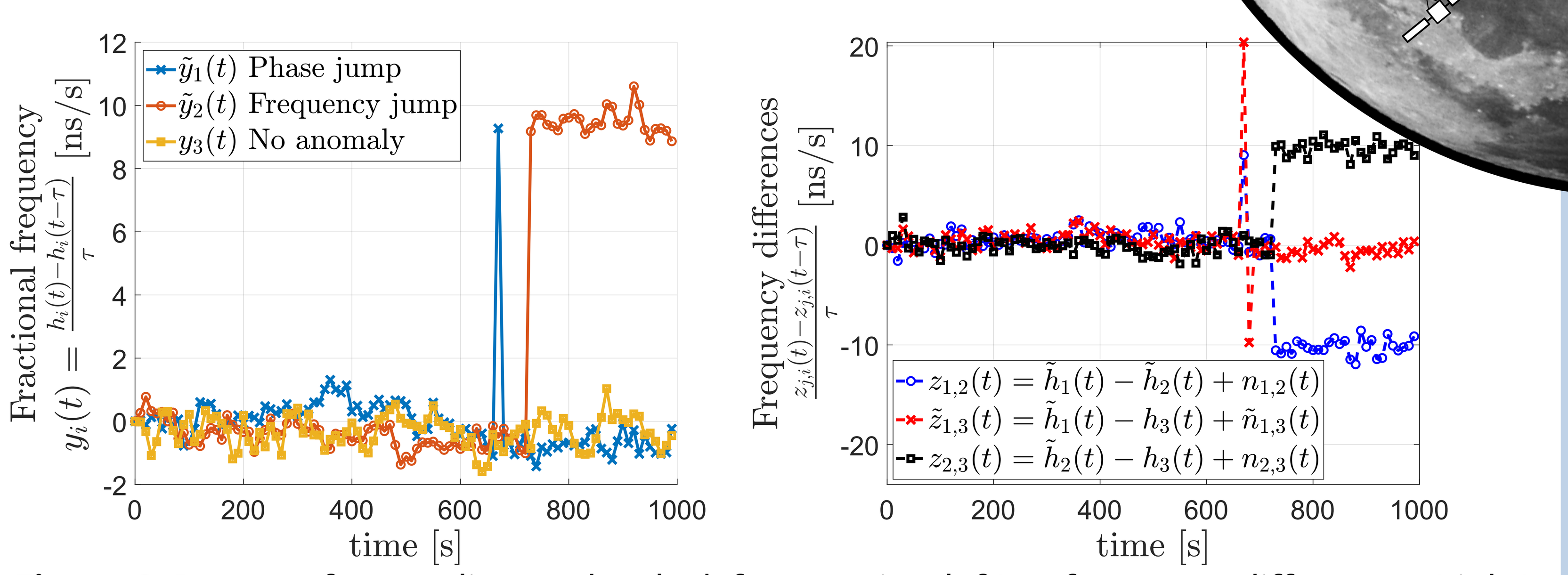
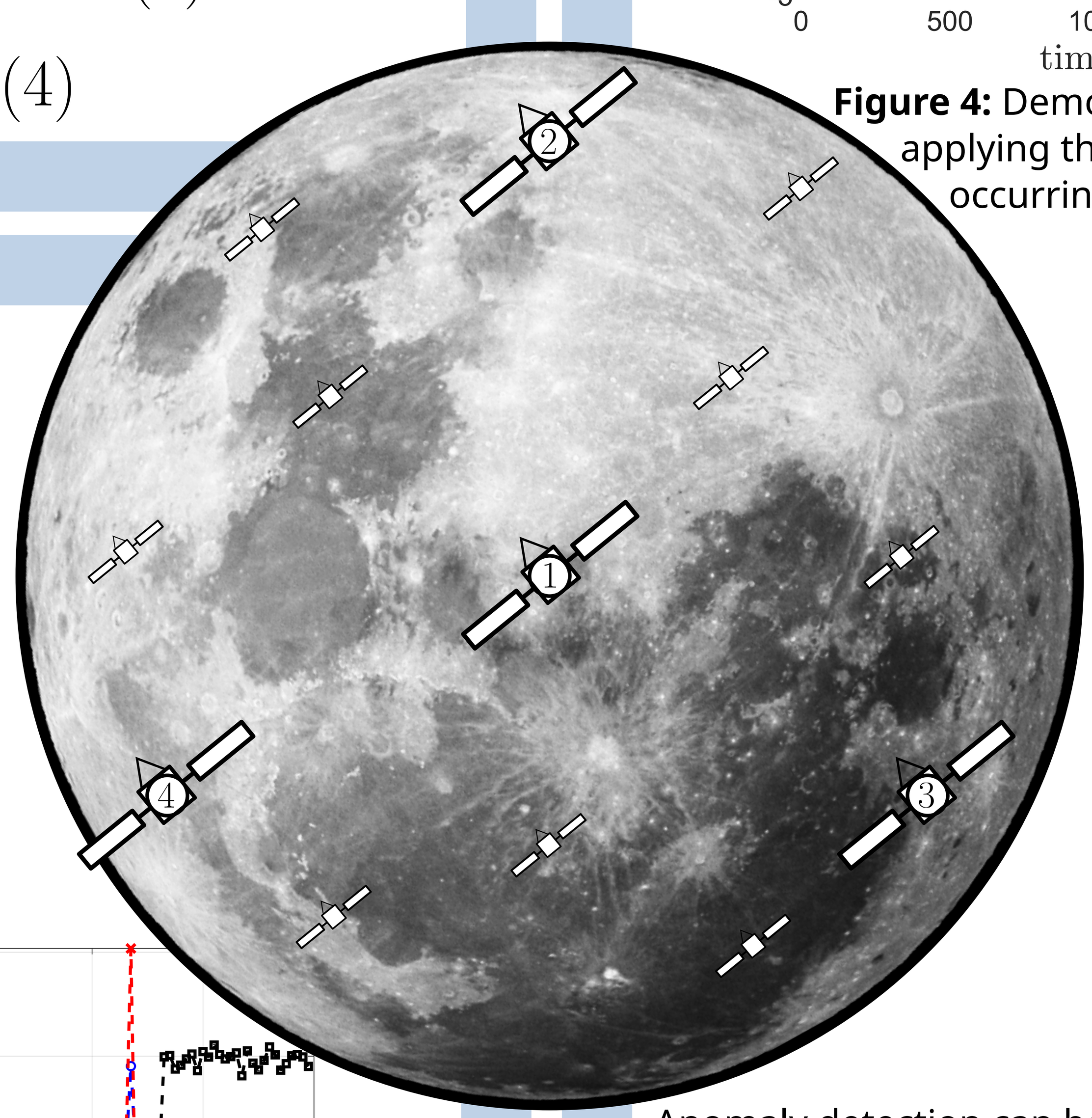


Figure 2: Impact of anomalies on the clock frequencies (left) or frequency differences (right).



4. Removal/Reintroduction of Isolated Clocks

Renormalizing the weights used in the BTSE after resetting the weights of isolated or newly regained clocks ensures continuity in the time scale with N_m missing clocks.

$$x_{i,E}(t_m)|_{N_m} = \sum_{j=1}^{N-N_m} w_j(t_m - \tau)\left[\hat{x}_{j,E}(t_m) - z_{j,i}(t_m)\right] \quad (8)$$

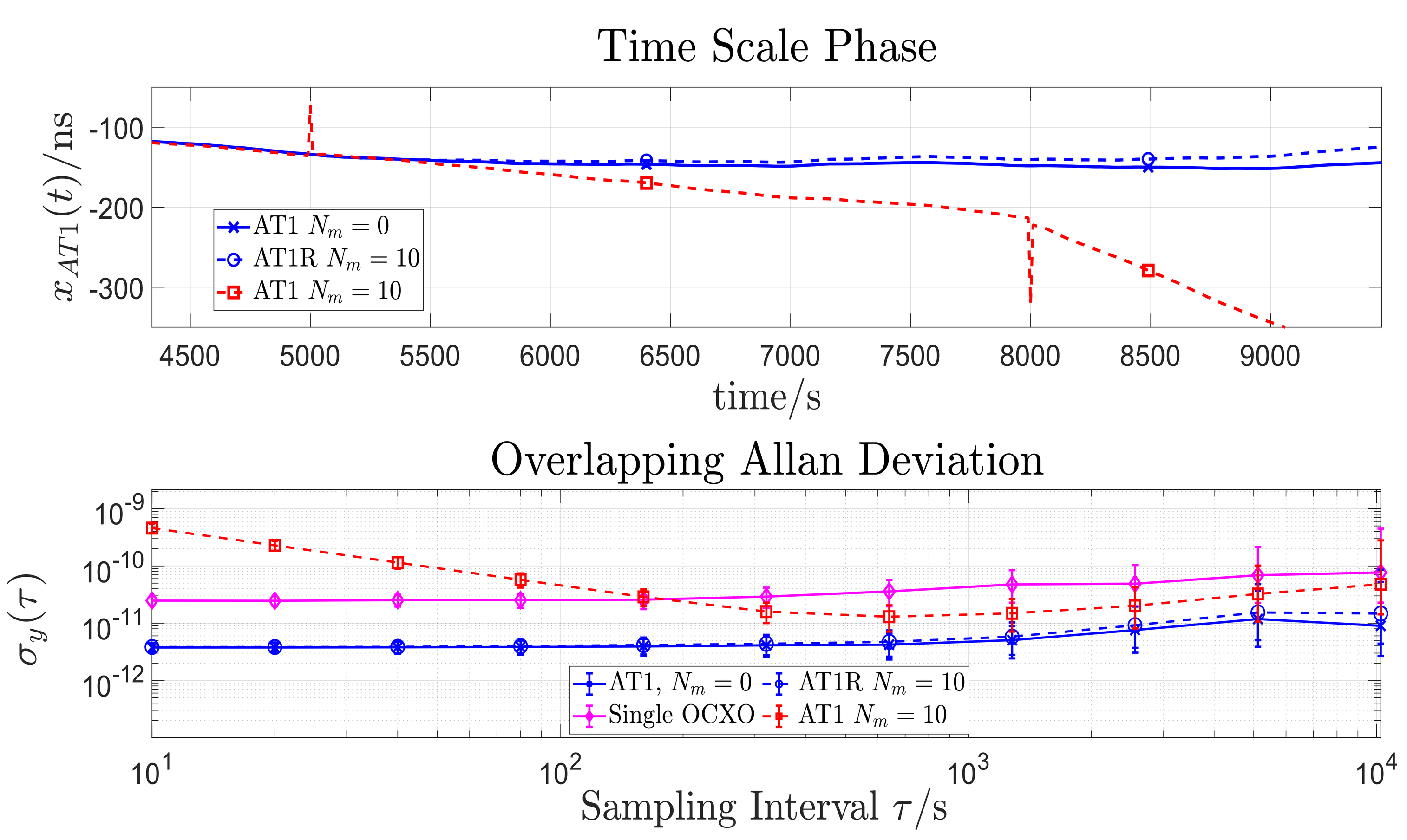


Figure 3: Time scale continuity and stability for 10 missing clocks during an outage period of 3000 s with and without the proposed reset methodology.

5. Missing Measurements

All measurements made between each unique pair of satellites can be presented as a linear combination of the N-1 time differences required to compute the BTSE for satellite 1:

$$\begin{bmatrix} z_{1,2}(t) \\ z_{1,3}(t) \\ z_{1,4}(t) \\ z_{2,3}(t) \\ z_{2,4}(t) \\ z_{3,4}(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_{1,2}(t) \\ x_{1,3}(t) \\ x_{1,4}(t) \end{bmatrix} + \begin{bmatrix} n_{1,2}(t) \\ n_{1,3}(t) \\ n_{1,4}(t) \\ n_{2,3}(t) \\ n_{2,4}(t) \\ n_{3,4}(t) \end{bmatrix} \quad (9)$$

The above can be written in a matrix form when all measurements are available (12) and in a reduced matrix form (13) that excludes the missing measurements.

$$\mathbf{z}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{n}(t) \quad \text{or} \quad \mathbf{z}_r(t) = \mathbf{A}_r(t)\mathbf{x}(t) + \mathbf{n}_r(t) \quad (10)$$

A Least Squares (LS) estimator reduces the impact of the measurement noise by estimating the required N-1 phase differences that minimize the summed square of the residuals.

$$\hat{\mathbf{x}}_{LS} = \min_{\mathbf{x}} \left\{ \|\mathbf{z} - \mathbf{A}\mathbf{x}\|^2 \right\} \longrightarrow \hat{\mathbf{x}}_{LS} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{z} \quad (11)$$

The achievable noise reduction for the LS estimator depends on the total number of measurements after removing outliers and considering unavailable inter-satellite links.

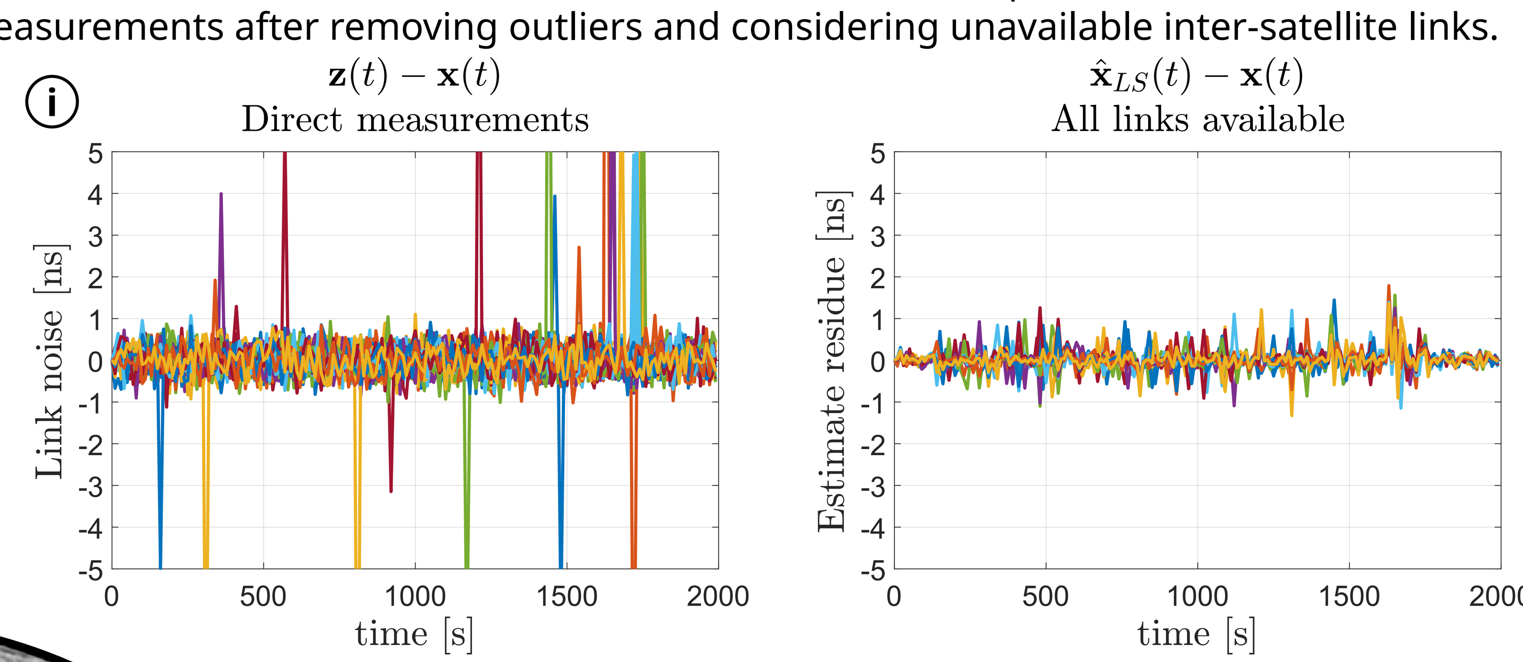


Figure 4: Demonstration of the equivalent noise remaining before and after applying the LS estimator on the N-1 measurements with anomalies occurring at different times.

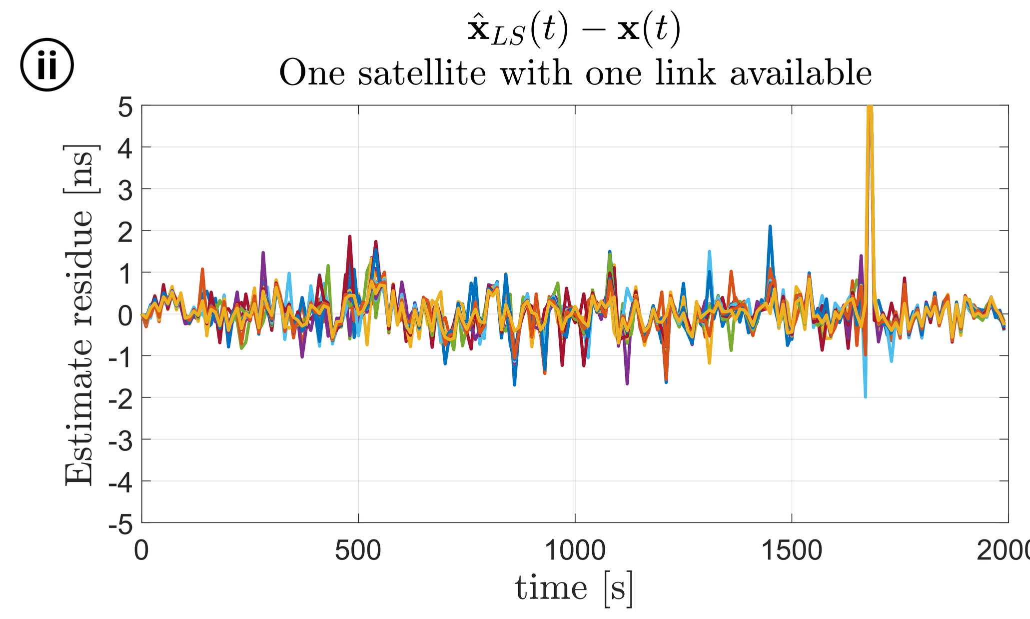


Figure 5: LS timing error when only one link is available.

Anomaly detection can be achieved using statistical methods [3]

Time scale algorithms can assign weights to be robust without detecting anomalies [4].

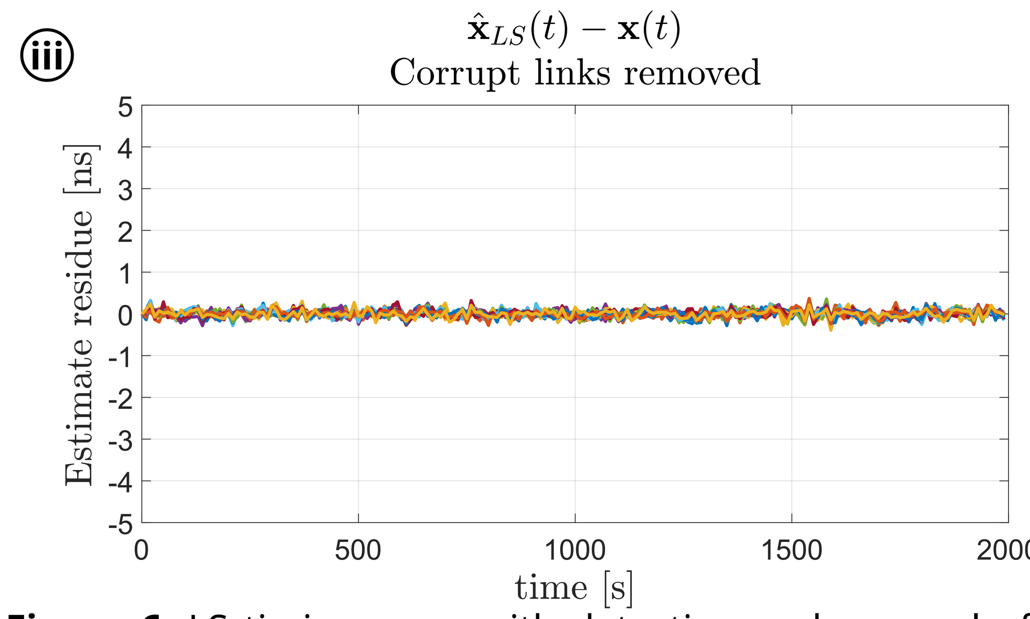


Figure 6: LS timing error with detection and removal of links with anomalies.

6. Conclusion and Future Work

- Inter-satellite clock bias measurements can be corrupted by either malfunctioning clocks or outliers in the measurement noise.
- The availability of the satellite clocks affects the stability of the time scale but can be dealt with by appropriately assigning weights.
- Measurement noise on the intersatellite links can be filtered by taking advantage of every unique measurement as a linear combination of the desired measurements.
- Dedicated detection methods should be investigated for identifying corrupted measurement links.
- Further work is required to determine the ideal smoothing time for reintroducing a missing clock into a robust time scale algorithm - Real time applications.

7. References

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