# **Multipath Estimating Techniques Performance Analysis**

Corentin Lubeigt TeSA/ISAE-SUPAERO 7, Boulevard de la Gare 31500 Toulouse, France corentin.lubeigt@tesa.prd.fr Lorenzo Ortega IPSA-Toulouse 40, Boulevard de la Marquette 31000 Toulouse, France lorenzo.ortega@ipsa.fr Jordi Vilà-Valls ISAE-SUPAERO 10, Avenue Édouard Belin 31055 Toulouse, France jordi.vila-valls@isae-supaero.fr

Laurent Lestarquit CNES 18, Avenue Édouard Belin 31401 Toulouse, France laurent.lestarquit@cnes.fr Eric Chaumette ISAE-SUPAERO 10, Avenue Édouard Belin 31055 Toulouse, France eric.chaumette@isae-supaero.fr

Abstract—In Global Navigation Satellite Systems, resilience to multipath remains an important open issue, being the limiting factor in several applications due to the environment specific nature of such harsh propagation conditions. In order to assess the multipath impact into the final system performance, accurate metrics are required. The multipath error envelope (MPEE), even if easy to handle, is limited to the study of the bias of a receiver architecture in a noise free environment. Moreover, when it is a flat zero-valued line, the MPEE becomes less informative about the parameter estimation performance. Considering an unbiased estimator, an alternative way to characterize an architecture is to evaluate its mean square error (MSE) and compare it to the corresponding Cramér-Rao bound (CRB). In this work, a methodology to use both aforementioned tools is presented. First, the MPEE, which is an understandable metric. Secondly, the MSE convergence to the CRB, where one can clearly interpret the estimation performance in terms of signal-to-noise ratio or minimum path separation. These tools are then applied to a range of known multipath mitigation techniques. In addition, a new alternating projection multipath mitigation approach is proposed and analyzed.

# **TABLE OF CONTENTS**

1. INTRODUCTION	1
2. FRAMEWORK OF THE STUDY	2
3. METHODOLOGY	2
4. RESULTS AND DISCUSSION	3
5. CONCLUSION	4
ACKNOWLEDGMENTS	5
REFERENCES	5
BIOGRAPHY	6

# **1. INTRODUCTION**

The design of new Global Navigation Satellite Systems (GNSS) architectures implies a trade-off between different performance criteria. Estimation accuracy, computational cost or robustness to multipath are examples of key GNSS receiver design criteria. Among them, resilience to multipath still remains an important open issue, indeed being the limiting factor in several applications due to the environment specific nature of such harsh propagation conditions. There are many ways to tackle the multipath issue: one is to use choke ring or specific antenna arrays to filter out the multipath, assuming that one knows its direction of arrival; another is to use signal processing techniques such

as Multipath Estimating Delay Lock Loop (MEDLL) [1] or Multipath Mitigation Technique (MMT) [2]; or a method of the family of algorithms which try to exploit the distortion of the correlation function, such as Vision Correlator [3], Pulse Aperture Correlator (PAC) [4], etc.

In order to assess the impact of possible multipath conditions into the final system performance, accurate metrics are required. From previous contributions, it is clear that, among many different tools [5], [6], [7], the de facto metric used to characterize the multipath effect is the so-called multipath error envelope (MPEE) [8], [9], [10]. The MPEE displays, for a given receiver architecture, the range of possible multipathinduced errors considering a simple two-ray model. This tool, however simple to handle, becomes less informative when it is a flat zero-valued line. In that case, the estimator under consideration is unbiased and thus its performance can no longer be characterized through the MPEE. In that case the estimator performance is only given by its variance, which in turn can only be evaluated in a noisy environment.

A way to characterize a given receiver architecture - provided the fact that it is unbiased - is to evaluate its mean square error (MSE) in the presence of noise for a number of signalto-noise (SNR) or path separation values and determine the threshold region where the MSE converges to the Cramér-Rao bound (CRB) that corresponds to the scenario under study. The CRB in a dual source context has been the object of several studies [11], [12]. In the latter, a handy closed-form expression was derived from the Slepian-Bangs formula [13] in the case of a narrowband signal. The CRB provides the best achievable performance of an unbiased estimator in terms of MSE. Then, it is meaningful to compare different architecture solutions according to their MSE when their corresponding MPEE is a flat zero, that is, when the estimator is unbiased. Notice that the CRB and the MPEE are complementary approaches.

In this work, a methodology is proposed to use both aforementioned tools: the MPEE which is an understandable metric, and the MSE convergence to the CRB where one can clearly read the limit of a technique in terms of SNR or minimum path separation. These tools are then applied to a range of existing (MEDLL, MMT, PAC) and new (Alternating Projection) multipath mitigation techniques. To support the discussion, a couple of well-known GNSS signals are compared in terms of both the MPEE and the MSE obtained through Monte Carlo simulations, namely, GPS L1 C/A and Galileo E1B signals. Notice that the former is the legacy GPS signal using a binary phase shift keying (BPSK) modulation with a large correlation function (the largest among the different GNSS signals in space), and the latter is built from a binary offset carrier (BOC) modulation, which implies a narrower correlation function but two sidelobes.

The outline of this work is as follows: in Section 2, the framework of this study is presented, including the signal model and a reminder on the different tools used. Section 3 presents the methodology, first introducing the algorithms under study, and then providing information on the simulations, the results of which are displayed and commented in Section 4. A concluding section closes this contribution.

# **2. FRAMEWORK OF THE STUDY**

## Signal Model

In this study, a simple two ray model is considered: a bandlimited signal s(t), modulated at a carrier frequency is received twice at the receiver's front-end: the first occurrence, indexed 0, is the line-of-sight (LOS) signal which has a given delay ( $\tau_0$ ) and complex attenuation (modulus  $\rho_0$  and phase  $\phi_0$ ). The second occurrence, indexed 1, is the single multipath, which is the result of a specular reflection from a reflecting object (e.g., the ground, a building, etc) and has its own delay ( $\tau_1$ ), and complex attenuation ( $\rho_1$  and  $\phi_1$ ). This multipath is assumed to arrive after the LOS signal:  $\tau_1 > \tau_0$ .

All front-end operations are done assuming that the Doppler frequency is known and compensated, as it is done in analyses involving MPEE [8]. The baseband output of the Hilbert filter with bandwidth *B* can be expressed as follows:

$$x(t) \triangleq \rho_0 e^{j\phi_0} s(t - \tau_0) + \rho_1 e^{j\phi_1} s(t - \tau_1) + w(t), \quad (1)$$

with w(t) an additive white Gaussian noise. Considering now the acquisition of  $N = N_2 - N_1$  samples at a sampling frequency  $F_s = 1/T_s$ , the discrete two-path conditional signal model (CSM) is,

$$\mathbf{x} = \mathbf{A}(\tau_0, \tau_1)\boldsymbol{\alpha} + \mathbf{w}, \ \mathbf{w} \sim C\mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N), \tag{2}$$

where  $\sigma_n^2 \mathbf{I}_N$  is the covariance matrix of the noise vector **w**,

$$\mathbf{A}(\tau_0, \tau_1) = [\mathbf{s}(\tau_0), \mathbf{s}(\tau_1)] \tag{3}$$

$$\mathbf{s}(\tau) = (\dots, s(nT_s - \tau), \dots)_{n \in [N_1, N_2]}$$
(4)

$$\mathbf{w} = (\dots, w(nT_s), \dots)_{n \in [N_1, N_2]}$$
(5)

$$\alpha^T = \left(\rho_0 e^{j\phi_0}, \rho_1 e^{j\phi_1}\right) \tag{6}$$

## Multipath Error Envelope

MPEE is a simple graphical tool that provides the range of the bias induced by a secondary source (the multipath) upon the estimation of the LOS signal time delay. For a given estimator, it can generally be defined as follows:

$$\max_{\Delta\phi} (b (MDR, \Delta\tau, \Delta\phi)), \min_{\Delta\phi} (b (MDR, \Delta\tau, \Delta\phi))$$
 (7)

where  $b(\cdot)$  is the induced bias on the estimation of the LOS delay, MDR =  $\rho_1/\rho_0$  is the multipath-to-direct ratio,  $\Delta \tau = \tau_1 - \tau_0$  is the excess delay of the multipath and  $\Delta \phi = \phi_1 - \phi_0$  is the relative phase. In practice, the bounds correspond when the multipath is in-phase ( $\Delta \phi = 0$ ) and out-of-phase ( $\Delta \phi = \pi$ ) of the LOS signal. As an example, Figure 1 presents the MPEE of the single source maximum likelihood estimator (1S-MLE) for two GNSS signals: GPS L1 C/A and GALILEO E1B and a precorrelation bandwidth set to 12 MHz.

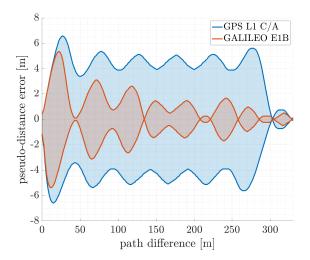


Figure 1. Example of MPEE for the single source maximum likelihood estimator for GPS L1 C/A and GALILEO E1B signals

# Cramér-Rao Bounds

The CRB describes the minimum variance achievable of an unbiased estimator. In [12], a closed-form CRB was derived for a dual source CSM that takes into account the contribution of the Doppler effect.

$$\mathbf{CRB}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}^{-1} = \mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}},\tag{8}$$

with  $\boldsymbol{\epsilon}^T = [\sigma_n^2, \tau_0, b_0, \rho_0, \phi_0, \tau_1, b_1, \rho_1, \phi_1]$  and b is the stretch induced by the Doppler effect.

To obtain the CRB that corresponds to the present model, that is for the unknown vector  $\boldsymbol{\zeta}^T = [\sigma_n^2, \tau_0, \rho_0, \phi_0, \tau_1, \rho_1, \phi_1]$ , one only needs to inject the fact that the Doppler frequency is assumed known and compensated. This can be done by adding the following constraint to the CSM studied in [12]:

$$b_0 = b_1 = 0 \Leftrightarrow \boldsymbol{\epsilon} = \boldsymbol{\epsilon}(\boldsymbol{\zeta}), \tag{9}$$

where, if one notes the 9-by-9 unity matrix  $\mathbf{I}_9 = [\mathbf{e}_1, \dots, \mathbf{e}_9]$  with  $\mathbf{e}_i$  the column unit vector with 1 at the *i*th component and 0 elsewhere, the constraint (9) simply becomes:

$$\boldsymbol{\epsilon}(\boldsymbol{\zeta}) = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_4 & \mathbf{e}_5 & \mathbf{e}_6 & \mathbf{e}_8 & \mathbf{e}_9 \end{bmatrix} \boldsymbol{\zeta} = \frac{\partial \boldsymbol{\epsilon}(\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}^T} \boldsymbol{\zeta},$$
(10)

and the CRB for the CSM (2) is obtained with the following matrix product:

$$\mathbf{CRB}_{\boldsymbol{\zeta}|\boldsymbol{\zeta}}^{-1} = \mathbf{F}_{\boldsymbol{\zeta}|\boldsymbol{\zeta}} = \left(\frac{\partial \boldsymbol{\epsilon}(\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}^T}\right)^T \mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}} \frac{\partial \boldsymbol{\epsilon}(\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}^T}.$$
(11)

# **3. METHODOLOGY**

Algorithms

In order to illustrate the combined use of MPEE and CRB, a set of four algorithms is investigated. Three of them are well known multipath mitigation algorithms:

• MEDLL as in [1] which consists of a relaxed version of a dual source CLEAN algorithm [14]. This algorithm estimates

a first source and then removes this estimate from the noisy samples in order to estimate a weaker source. It iterates until the associate likelihood does not change anymore.

• PAC [4] belongs to the family of algorithms known as double-delta ( $\Delta\Delta$ ). Those algorithms improve with a limited number of extra correlators the basic early-minus-late architecture. PAC estimates the slope at each side of the correlation function peak and if there is any asymmetry, the algorithm compensates for it by assuming that a single multipath is causing this asymmetry. In this implementation, the spacing was set to 1/12 of L1 C/A chips.

• MMT [2] is an implementation of the dual source maximum likelihood estimator (2S-MLE). This estimator has particular properties that make it a benchmark for the other algorithms: it has been shown that in the asymptotic regime, when the SNR is large enough [15], the 2S-MLE is efficient, i.e., it is unbiased and its variance is equal to the corresponding CRB.

Along with these three algorithms, an implementation of the Alternating Projection Estimator (APE) [16] is proposed in this study. This dual source implementation simplifies the 2S-MLE multi-dimensional parameter search to one-dimension by introducing an adequate projector. Thus, it iteratively maximizes the likelihood criterion with respect to the first source and then to the second source.

## Variables of Interest

Traditionally, the MPEE represents the bias induced by a multipath as a function of the path separation between the LOS signal and the multipath, i.e.,  $\Delta \tau$ . It is evaluated for the in-phase and out-of-phase cases with a fixed MDR and with an infinite SNR. Consequently, a first variable that will drive the following study is this relative time delay  $\Delta \tau$ .

On the other hand, in this study, the aim is to investigate the behavior of the estimator at finite SNR. Then, it is necessary to display the MSE of the algorithms for different SNR values. Thus, the second variable of interest is the SNR at the output of the matched filter that can be defined as:

$$SNR_{out} \triangleq \frac{\rho_0^2}{\sigma_n^2} \int_0^{T_I} |s(t)|^2 dt$$
 (12)

where  $T_I$  is the correlation integration time.

#### Simulation Set-up

The next section presents for each algorithm and for two well-known GNSS signals, GPS L1 C/A ( $T_I^{\text{GPS}} = 1 \text{ms}$ ) and GALILEO E1B ( $T_I^{\text{GAL}} = 4 \text{ms}$ ) with a pre-correlation bandwidth  $B = F_s$  set to 12 MHz and a MDR set to 0.5:

the corresponding MPEE with regard to the path separation,
the MSE with regard to the path separation for a fixed value of SNR<sub>out</sub> (GPS L1 C/A: SNR<sub>out</sub> = 31dB and GALILEO E1B: SNR<sub>out</sub> = 34dB),

• and the MSE with regard to the SNR<sub>out</sub> for a fixed value of  $\Delta\tau$  (set to 0.5 C/A chips or about 150m).

MSE were evaluated from 1000 Monte Carlo runs. For these particular examples, the relative phase between the LOS and the multipath was set to  $\Delta \phi = 0$ .

# **4. RESULTS AND DISCUSSION**

# MPEE

Figure 2 and Figure 3 present the MPEE for the algorithms under study applied to GPS L1 C/A and GALILEO E1B signals. Results are obtained by simply observing the output of the estimators in the presence of a single multipath without noise. A first result that can be drawn from these figures is that the PAC MPEE (in magenta), however small, never reduces to zero. Consequently, it can be said of this architecture that the presence of a multipath will irremediably affect the precision of the LOS time delay. On the other hand, MEDLL (in blue) and APE (in orange) MPEEs both present interesting behaviors. There is a path separation threshold above which these algorithms estimate the LOS time delay with no bias. Note that for GPS L1 C/A, the MEDLL and APE MPEEs thresholds are around 140m and 40m, respectively. For GALILEO E1B, both algorithms present a similar threshold at 40m. As expected, the MMT/2S-MLE does not present any bias, as this method correctly estimates the two sources in a noiseless environment.

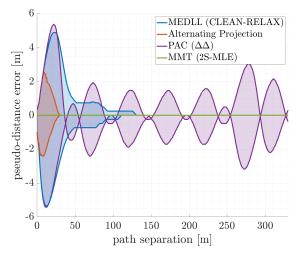


Figure 2. MPEE for GPS L1 C/A.

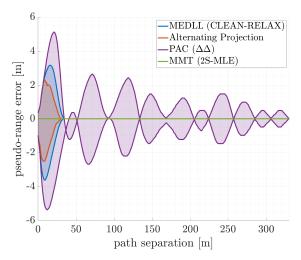


Figure 3. MPEE for GALILEO E1B.

## MSE with Regard to the Path Separation

In order to complete the previous experience, noise is included at the input of the receiver. Figures 4 and 5 illustrate the MSE of the LOS signal time delay estimate as a function of the path separation. Moreover, the CRB (black solid line) (11) is also illustrated. When the path separation is close to zero, the dual source model CRB for the estimation of the LOS time delay naturally soars to infinity since it becomes impossible to identify the LOS signal from the multipath signal. In this region all the estimators behaves better than the CRB because only one source is observed.

Except for the PAC MSE, which is altered by a non-zero bias, it can be observed that the other three algorithms satisfyingly reach the CRB at a given path separation. For GPS L1 C/A, when MEDLL is used, the MSE reaches the CRB at about 100m of path separation, which is a little before becoming fully unbiased according to its MPEE. On the other hand, it is noteworthy to remark that in Figure 2, between 100m and 140m the upper bound of the MPEE, which corresponds to the in-phase case, is at about 0.2m. However, in Figure 4 the resulting root MSE (RMSE) around this area is at about 1m. That involves that the bias observed from the MPEE is masked by a larger value of RMSE. The APE MSE coherently reaches the CRB at about 40m as hinted by its MPEE.

Finally, the MMT estimator was supposed to stick to the CRB at any path separation. However, it does not behave so in the range between 0 to 40m due to the practical implementation, i.e., the search area for  $\tau_0$  and  $\tau_1$  was limited for computational load reasons and it did not allow large output values. In short, in this range of path separation, the estimation of the time delays was helped by the limited search area.

Similar comments apply for Figure 5.

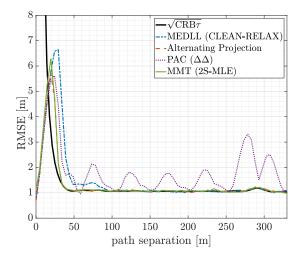
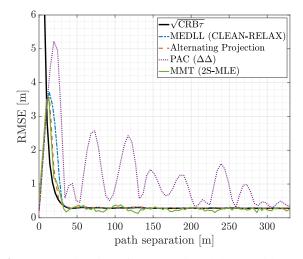


Figure 4. Estimation of the LOS time delay  $\tau_0$  with respect to the path separation for GPS L1 C/A at SNR<sub>out</sub> = 31dB.

## MSE with Regard to the SNR

It is well-known that for low SNR, an algorithm might not be able to detect a signal over the thermal noise. Then, there is a strong interest to understand how the considered algorithms behave when the SNR varies. The main feature that one might look for is, for a given path separation, the SNR threshold, i.e., the minimum SNR level necessary to detect the signal in an efficient way.



**Figure 5**. Estimation of the LOS time delay  $\tau_0$  with respect to the path separation for GALILEO E1B at SNR<sub>out</sub> = 34dB.

Figures 6 and 7 illustrate the MSE for each algorithm and for both GNSS signals with respect to the SNR. Both figures can be divided into three areas: in the left-hand side the SNR is so small that any estimator is outputting meaningless random estimates; then when the SNR rises, there is a transition area in which the MSE is not yet reaching the CRB but is not out of range. Finally, in the right-hand side, there is a particular operation point where the SNR is large enough for the estimators to behave efficiently. It is exactly that transition point that is referred to as SNR threshold.

From Figure 6, the considered estimators can be compared according to their behaviour: the threshold is slightly better for the MMT algorithm ( $SNR_{out}=21dB$ ), followed by the APE ( $SNR_{out}=22dB$ ) and the MEDLL ( $SNR_{out}=24dB$ ). The MSE of the PAC algorithm (in magenta) does not seem to reach the CRB. Note that this is due to the fact that a biased estimator does not necessarily have a MSE that follows the CRB. Indeed at path separation of 150m, from Figure 2, one can read that an in-phase multipath induces a bias of about -0.2m. This phenomenon is particularly visible in the case of the GALILEO signal, in Figure 7, where the PAC RMSE converges to a constant value of about 1.5m, which is exactly the value of the PAC upper bound MPEE for this specific path separation of 150m (refer to Figure 3).

# **5.** CONCLUSION

In this contribution, a set of multipath mitigating estimators was analyzed not only with the well-known MPEE tool but also with the CRB, which takes into account noisy environments and provides informative features on the MSE. This approach coherently completes the MPEE tool, emphasizing the impact of a bias on the performance and giving a way to compare algorithms even when their MPEE converges to zero (no bias). The entire analysis can provide the following features that are easy to extract from the results and that are key for the comparison of different algorithms:

- bias from the MPEE,
- variance lower bound from the CRB,
- threshold with regard to the path separation from which the
- MSE reaches the CRB for a given SNR,
- threshold with regard to the SNR from which the MSE

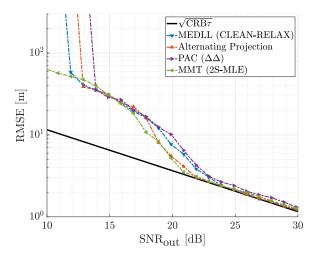
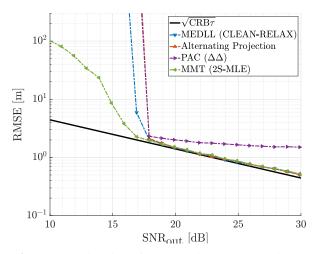


Figure 6. Estimation of the LOS time delay  $\tau_0$  with regard to SNR<sub>out</sub> for GPS L1 C/A and path separation of 150m.



**Figure 7**. Estimation of the LOS time delay  $\tau_0$  with regard to SNR<sub>out</sub> for GALILEO E1B and path separation of 150m.

reaches the CRB for a given path separation.

Finally, through this analysis, an implementation of the Alternating Projection estimator has been first presented. Moreover, it has been shown, at least for the set of scenarios studied here, that the Alternating Projection technique can be a good candidate for multipath mitigation.

## ACKNOWLEDGMENTS

This work has been partially supported by the DGA/AID project 2019.65.0068.00.470.75.01 and the French Space Agency (CNES).

# REFERENCES

P. C. [1] B. R. Townsend, Fenton, K. J. Van Van Dierendonck, and D. J. Richard Nee. "Performance Evaluation of Multipath the Estimating Delay Lock Loop," Navigation, vol. 42, no. 3, pp. 502–514, 1995. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/j.2161-4296.1995.tb01903.x

- [2] L. R. Weill, "Multipath Mitigation using Modernized GPS Signals: How Good Can it Get?" Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), pp. 493–505, September 2002.
- [3] P. C. Fenton and J. Jones, "The Theory and Performance of NovAtel Inc.'s Vision Correlator," *Proceedings of the 18th International Technical Meeting of the Satellite Division of The Institute of Navigation*, pp. 2178–2186, January 2005.
- [4] J. Jones, P. Fenton, and B. Smith, "Theory and Performance of the Pulse Aperture Correlator," NovAtel Inc., Tech. Rep., 2004. [Online]. Available: https://hexagondownloads.blob.core.windows.net /public/Novatel/assets/Documents/Papers/PAC/PAC.pdf
- [5] C. Hegarty, M. Tran, and J. W. Betz, "Multipath Performance of the New GNSS Signals," in *Proceedings of the 2004 National Technical Meeting of The Institute of Navigation*, 2004, pp. 333–342.
- [6] M. Brenner, R. Reuter, and B. Schipper, "GPS Landing System Multipath Evaluation Techniques and Results," in *Proceedings of the 11th International Techni*cal Meeting of the Satellite Division of the Institute of Navigation (ION GPS 1998), 1998, pp. 999–1008.
- [7] M. Irsigler, J. A. Avila-Rodriguez, and G. W. Hein, "Criteria for GNSS Multipath Performance Assessment," in *Proceedings of the 18th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2005)*, 2005, pp. 2166–2177.
- [8] E. Kaplan and C. Hegarty, Understanding GPS/GNSS: Principle and Applications, 3rd ed. Artech House, 2017.
- [9] P. J. G. Teunissen and O. Montenbruck, Eds., *Handbook* of Global Navigation Satellite Systems. Switzerland: Springer, 2017.
- [10] Y. J. Morton, F. Van Diggelen, J. J. Spilker Jr, B. W. Parkinson, and S. L. G. Gao, *Position, Navigation, and Timing Technologies in the 21st Century, Volumes 1 and 2.* John Wiley & Sons, 2020.
- [11] M. A. Ribot, C. Botteron, and P. Farine, "Derivation of the Cramér-Rao Bound in the GNSS-Reflectometry Context for Static, Ground-Based Receivers in Scenarios with Coherent Reflection," *Sensors*, vol. 16, no. 12, p. 2063, 2016. [Online]. Available: https://doi.org/10.3390/s16122063
- [12] C. Lubeigt, L. Ortega, J. Vilà-Valls, L. Lestarquit, and E. Chaumette, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," *Remote Sensing*, vol. 12, no. 23, 2020. [Online]. Available: https://www.mdpi.com/2072-4292/12/23/3894
- [13] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory. Englewood Cliffs, New Jersey, USA: Prentice-Hall, 1993.
- [14] J. Li and P. Stoica, "Efficient mixed-spectrum estimation with applications to target feature extraction," *IEEE Transactions on Signal Processing*, vol. 44, no. 2, pp. 281–295, 1996.
- [15] A. Renaux, P. Forster, E. Chaumette, and P. Larzabal, "On the High-SNR Conditional Maximum-Likelihood Estimator Full Statistical Characterization,"

*IEEE Trans. Signal Process.*, vol. 54, no. 12, pp. 4840 – 4843, Dec. 2006.

[16] I. Ziskind and M. Wax, "Maximum Likelihood Localization of Multiple Sources by Alternating Projection," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 10, pp. 1553–1560, 1988.

# BIOGRAPHY



**Corentin Lubeigt** is at the Télécommunications pour le Spatial et l'Aéronautique laboratory (TéSA) and the Institut Supérieur de l'Aéronautique et de l'Espace (ISAE-SUPAERO), University of Toulouse, France. He received the MS in Engineering at ISAE-SUPAERO in 2018 and is currently preparing a PhD in signal processing for GNSS-R.



Lorenzo Ortega is Associate Profesor at Institut Polytechnique des Sciences Avancées (IPSA), Toulouse, France. He received the MS in Electrical Engineering from Zaragoza University, Spain, in 2016 and PhD in Signal Processing from National Polytechnic Institute of Toulouse (INPT) in 2019. From 2020 to 2021, he was a postdoctoral researcher at the Institut Supérieur de

l'Aéronautique et de l'Espace (ISAE-SUPAERO), University of Toulouse, France. His primary areas of interest include statistical signal processing, machine learning, estimation and detection theory, channel coding and digital communications, with applications to satellite communication, localization, tracking, navigation and remote sensing.



Jordi Vilà-Valls (S'08 – M'13 – SM'17) is Associate Professor at the Institut Supérieur de l'Aéronautique et de l'Espace (ISAE-SUPAERO), University of Toulouse, France. He received the MS in Electrical Engineering from both Universitat Politècnica de Catalunya (UPC), Spain, and Grenoble Institute of Technology (INPG), France, in 2006, and the PhD in Signal Processing from

INPG in 2010. His primary areas of interest include statistical signal processing, estimation and detection theory, nonlinear Bayesian inference and robust filtering, with applications to localization, tracking, navigation and remote sensing. He is Elected Member of the EURASIP TAC on Theoretical and Methodological Trends in Signal Processing, AE for Signal Processing, and has been (is) actively involved in the organizing committees of EUSIPCO (2019, 2021, 2024) and IEEE CAMSAP (2019, 2023).



Laurent Lestarquit is a GNSS signal expert, at the French space agency (CNES) in Toulouse. He is currently involved in research in the field of GNSS scientific use for earth and atmosphere remote sensing and GNSS signal metrology. He graduated from the Ecole Polytechnique, Paris, France, in 1994 and received his space engineering diploma from Supaero (now ISAE) in 1996. He received

the 2017 European Inventor Award from the European Patent Office for the invention of the Alt-BOC modulation and the CBOC waveform now used in GALILEO.



**Eric Chaumette** was born in 1965 at Chartres (France). He studied Electronics and Signal Processing both at ENAC (Toulouse, France) where he obtained an Engineer degree in 1989, and at Toulouse University where he obtained a M.Sc. degree in Signal Processing in 1989. From 1990 to 2007, he was with Thales in various radar studies departments. From 2007 to 2013, he was

with the Electromagnetic and Radar Division of the French Aerospace Lab (ONERA), Palaiseau, France, as a research engineer. Simultaneously, from 2000 to 2014, he was a research associate at laboratory SATIE, CNRS, École Normale Supérieure de Cachan, France, where he received the PhD degree in 2004 and the "habilitation à diriger les recherches" in 2014. He is currently Professor at the Department of Electronics, Optronics and Signal of ISAE-SUPAERO, Toulouse, France. Main domains of interest are related to detection and estimation theory applied to localisation and navigation (GNSS, robust multi-sensor data fusion).