Synchronization, detection and equalization for Continuous Phase Modulation over doubly-selective channels

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CENTRE NATIONAL D'ÉTUDES SPATIALES

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Telecommunications for Space and Aeronautics

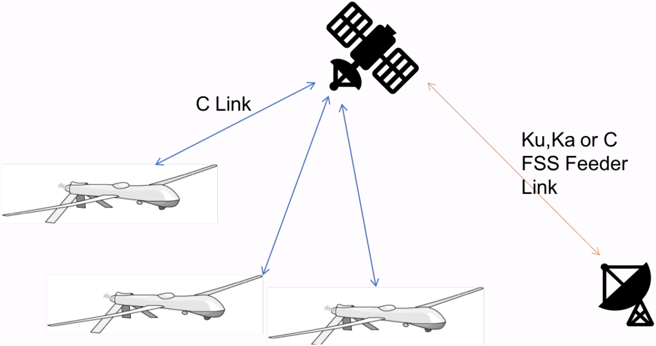
Context

- Unmanned Aerial Vehicule (*UAV*)
  - Earth observation
  - Remote sensing
  - Communications
  - Entertainment
  - Goods delivery
- Must ensure the reliability of the communication system with the UAV

Figure: Spy'Ranger © Aviation Design 2017

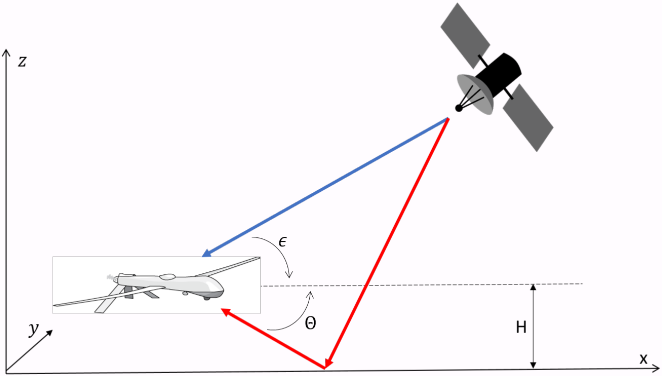
Context

- Command & Non Payload Communication Link by Satellite



Constraints

- Non linearities introduced by embedded amplifiers
- Multi-path channels



Choice of Continuous Phase Modulation

- Non-linear modulation;
- Constant complex envelope;
- Robustness to non-linearities introduced by amplifiers (no need of IBO);
- Good spectral occupancy;
- already in use in satellite communication standards (such as DVB-RCS2) or for tactical communications.

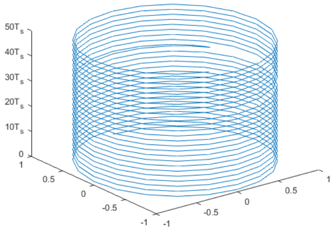


Figure: CPM signal in a polar plan (Quaternary CPM, L=2, h=1/4, RC pulse shape

Main contributions

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- A new Minimum Mean Square Error - Frequency Domain Equalizer for CPM transmissions over frequency-selective channels
  - Has the same performance as others State of the Art Equalizers
  - But with a significantly lower computational complexity
  - Has been extended to a low-complexity "approximate" equalizer in case of doubly-selective channels

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- A new Minimum Mean Square Error - Frequency Domain Equalizer for CPM transmissions over frequency-selective channels
  - Has the same performance as others State of the Art Equalizers
  - But with a significantly lower computational complexity
  - Has been extended to a low-complexity "approximate" equalizer in case of doubly-selective channels
- Joint Carrier Frequency Offset and channel estimation
  - Compatible with the equalization schemes for CPM
  - Reaches asymptotically the Cramér Rao Bound
  - can use a parametric model (case of the aeronautical channel)
  - Has been extended to Time-Varying channels

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BICM for CPM

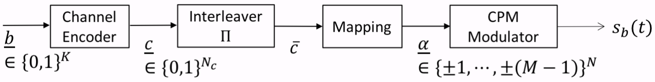

Figure: BICM for CPM signals

Complex envelope

Complex envelope $s_b(t)$ associated with the transmitted CPM signal:

$$s_b(t) = \sqrt{\frac{2E_s}{T_s}} \exp\left(j\theta(t, \underline{\alpha})\right) \tag{1}$$

$$\text{where } \theta(t, \underline{\alpha}) = 2\pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT_s)$$

$$\text{and } q(t) = \begin{cases} \int_0^t g(\tau) d\tau, t \leq LT_s \\ 1/2, t > LT_s \end{cases}$$

E_s is the symbol energy, T_s is the symbol period, $\theta(t, \underline{\alpha})$ is the information phase, $g(t)$ is the pulse response, h is the modulation index and L is the CPM memory.

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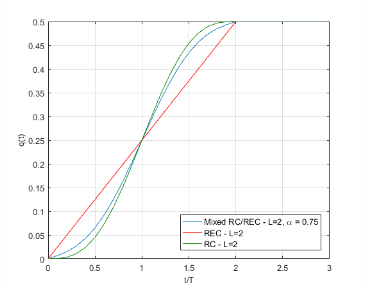
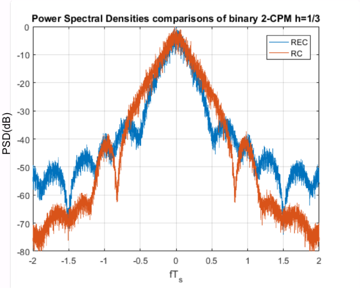
CPM parameters

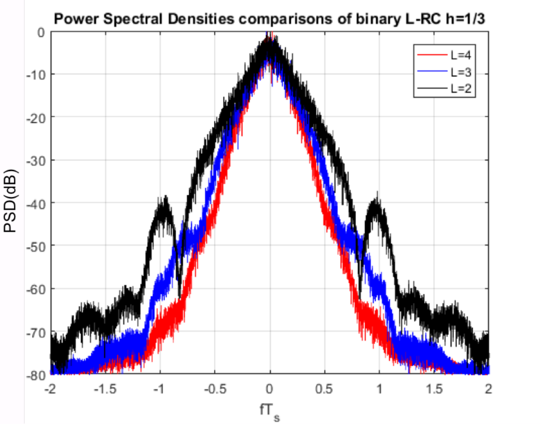
- Pulse shapes
  - Gaussian Minimum Shift Keying (*GMSK*)
  - Rectangular (*REC*)
  - Raised Cosine (*RC*)
  - ...
- with different properties on the CPM signal





CPM parameters

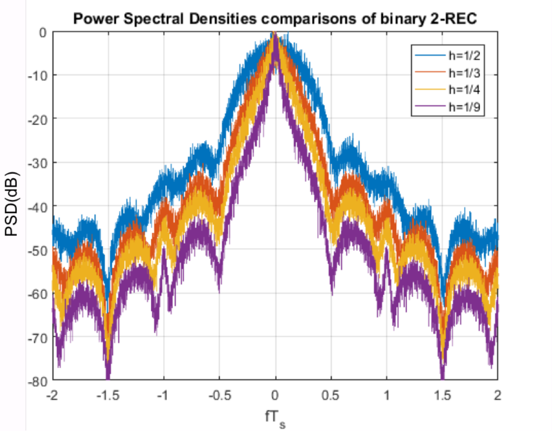
- CPM Memory L



- When L increases
  - Reduces the spectrum occupancy
  - Increases the receiver complexity

CPM parameters

- Modulation index h
  - h is generally a rational number $h = \frac{k}{p}$ smaller than 1
  - p has an influence on the number of state of the CPM trellis
  - When h decreases, the bandwidth occupancy is smaller ... but the Euclidean distance is smaller too



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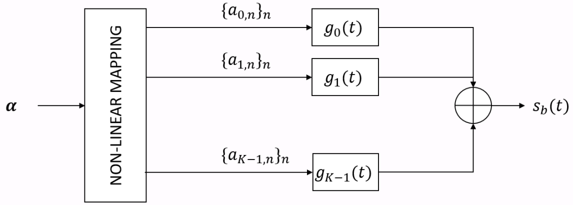
Laurent Decomposition

- A binary CPM with non-integer modulation index can be represented as a sum of linear PAM [Lau86]:

$$s_b(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_{k,n} g_k(t - nT_s) \tag{2}$$

- K is the number of Laurent Pulses
- The non-linearities is within the pseudo-symbols $\{a_{k,n}\}$
- The PAM decomposition has been extended to M-ary CPMs and also to integer indices schemes



Features of PAM Decomposition

- Most of the energy is within the first "M-1" components

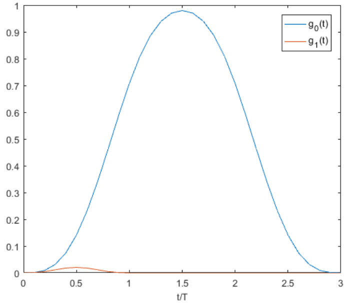


Figure: Example: Binary CPM, Averaged REC/RC ($\alpha = 0.75$), $L = 2$ and $h = 1/2$

- Can be used to design low-complexity detector

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CPM Detector (AWGN Channel)

- Received signal: $r(t) = s_b(t) + w(t)$

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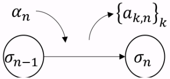

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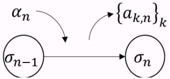

- Sufficient statistics:

$$r_{k,n} = \int_0^{(L+1)T_s} r(t + nT_s) g_k^*(-t) dt \qquad (3)$$

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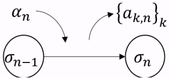

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- Branch metric:

$$\mathcal{G}_n(\sigma_{n-1}, \sigma_n) \propto \exp\left\{ \frac{2}{N_0} \Re\{ \sum_{k=0}^{K-1} r_{k,n} a_{k,n}^* \}) \right\} \pi(\alpha_n) \tag{4}$$

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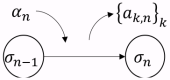

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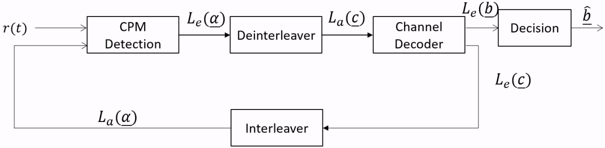
- Low Complexity Design by taking $K = M - 1$

Overall Receiver over AWGN channels

- Iterative receiver



- Non-iterative receiver
  - No iteration between the CPM detection and the outer channel decoder
  - Can result in a non-negligible loss of performance
  - May be optimal in case of "non-recursive" CPM or in case of "pragmatic CPM" (precoder)

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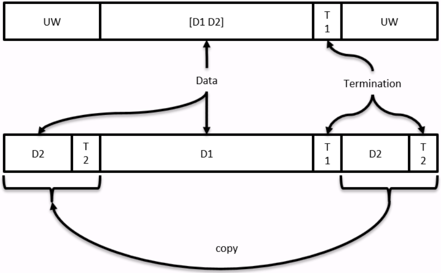
Contributions

- Equivalences and differences between State of the Art MMSE-FDE for CPM over frequency-selective channels
- Design of a low-complexity MMSE-FDE for CPM over frequency-selective channels
- Extension to CPM transmissions over doubly selective channels

System Model

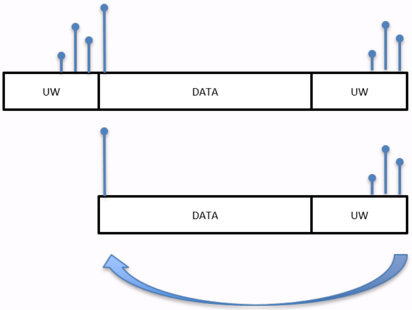
- Received signal is the linear convolution between the transmitted signal and the channel (filtered by a LPF)
- Circularization of the signal by the insertion of a CP or an UW
  - Similar to linear modulation up to termination symbols to ensure the phase continuity



System Model

- Circular convolution between the channel and the transmitted over-sampled complex envelop

$$r[n] = \sum_{m} h[m]s[\mathrm{mod}(n-m, kN)] \tag{5}$$



MMSE-FDE for CPM over TIV channels

- Optimal receiver
  - Joint channel equalization and data detection
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  - Channel and Laurent Pulses Equalizer (Pancaldi)
  - Channel Equalizer (Van Thillo)
  - Equivalence

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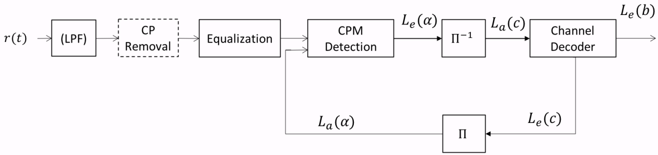
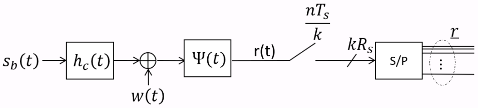
  - Equivalence
- Contribution
  - A new exact low complexity MMSE-FDE
  - Complexity in O(Nlog(N))
  - Same performance as the others equalizers

Receiver Structure



Channel and Laurent Pulses Equalizer (1/2)

- Fractionally-spaced representation of the signal



- $\Psi(t)$ ideal LPF
- Received signal in the FD:

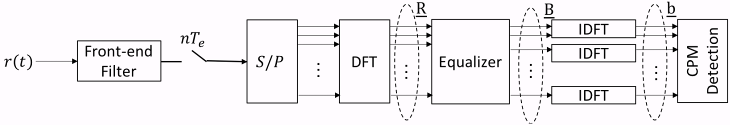
$$\underline{\boldsymbol{R}} = \underbrace{\underline{\underline{\boldsymbol{H}}}\,\underline{\underline{\boldsymbol{L}}}}_{\triangleq \underline{\underline{\boldsymbol{M}}}}\,\underline{\boldsymbol{B}}_{2N} + \underline{\boldsymbol{W}} = \underline{\underline{\boldsymbol{M}}}\,\underline{\boldsymbol{B}}_{2N} + \underline{\boldsymbol{W}} \qquad (6)$$

- Equalizer given by $\underline{\underline{\boldsymbol{D}}}_{\mathrm{LE}} = \underline{\underline{\boldsymbol{J}\boldsymbol{M}}}^H[\underline{\underline{\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{M}}}^H + \sigma_n^2 I_{2N}]^{-1}$:

$$\widehat{\underline{\boldsymbol{B}}} = \underline{\underline{\boldsymbol{J}\boldsymbol{M}}}^H[\underline{\underline{\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{M}}}^H + \sigma_n^2 I_{2N}]^{-1}\underline{\boldsymbol{R}} \qquad (7)$$

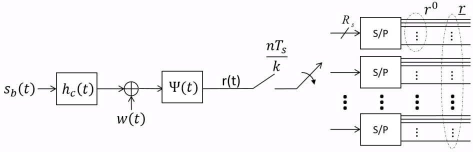
Channel and Laurent Pulses Equalizer (2/2)

- Interest:
  - If only one LP is considered, similar to [TS05]
  - Similar to a Fractionally Spaced Equalizer (taking into account the correlation of the pseudo-symbols)
- Main issues:
  - Full auto-correlation matrix of the pseudo-symbols vectors $\underline{\underline{\mathbf{\Phi}}}$ to inverse
  - Requires a non-conventional CPM detector



Channel Equalizer (1/2)

- Polyphase representation of the signal



- Received signal in the FD:

$$\underline{\boldsymbol{R}}_p = \underbrace{\underline{\underline{\boldsymbol{H}}}_p \underline{\underline{\boldsymbol{L}}}_p \underline{\boldsymbol{B}}}_{\triangleq \underline{\boldsymbol{S}}_p} + \underline{\boldsymbol{W}}_p = \underline{\underline{\boldsymbol{H}}}_p \underline{\boldsymbol{S}}_p + \underline{\boldsymbol{W}}_p \qquad (8)$$

- Equalizer given by

$$\underline{\underline{\boldsymbol{G}}} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}} \underline{\underline{\boldsymbol{H}}}_p^H [\underline{\underline{\boldsymbol{H}}}_p \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}} \underline{\underline{\boldsymbol{H}}}_p^H + \sigma_n^2 \underline{\underline{\boldsymbol{I}}}_{2N}]^{-1}$$

$$\widehat{\underline{\boldsymbol{S}}}_p = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}} \underline{\underline{\boldsymbol{H}}}_p^H [\underline{\underline{\boldsymbol{H}}}_p \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}} \underline{\underline{\boldsymbol{H}}}_p^H + \sigma_n^2 \underline{\underline{\boldsymbol{I}}}_{2N}]^{-1} \underline{\boldsymbol{R}}_p \qquad (9)$$

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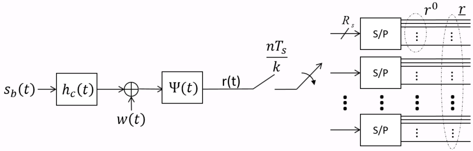

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Channel Equalizer (2/2)

- $\underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}}$ non diagonal matrix
  - Due to the polyphase representation, its Time-Domain counterpart is NOT a circulant matrix:

$$\underline{\underline{\boldsymbol{r}}}_{\mathbf{ss,p}} = \mathbb{E}[\underline{\boldsymbol{s}}_p \underline{\boldsymbol{s}}_p^H]$$

$$= \begin{bmatrix} r_{\mathbf{ss}}(0) & r_{\mathbf{ss}}^*(2) & r_{\mathbf{ss}}^*(4) & \dots & r_{\mathbf{ss}}^*(2N-2) & r_{\mathbf{ss}}^*(1) & r_{\mathbf{ss}}^*(3) & \dots & r_{\mathbf{ss}}^*(2N-1) \\ r_{\mathbf{ss}}(2) & r_{\mathbf{ss}}(0) & r_{\mathbf{ss}}^*(2) & \dots & r_{\mathbf{ss}}^*(2N-4) & r_{\mathbf{ss}}(3) & r_{\mathbf{ss}}^*(1) & \dots & r_{\mathbf{ss}}^*(2N-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

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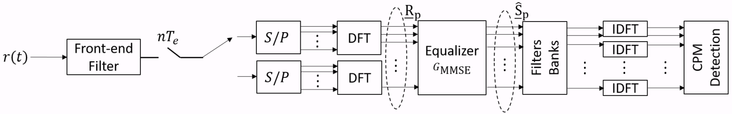
- Conventional CPM Detector



Equivalence

- Both MMSE-FDE (same sampling rate...)
- Should have the same performance
- Channel Equalizer: $\underline{\widetilde{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{G}}}\,\underline{\boldsymbol{R}}$
- Channel and LP Equalizer: $\underline{\widehat{\boldsymbol{B}}} = \underline{\underline{\boldsymbol{D}}}_{\mathrm{LE}}\underline{\boldsymbol{R}}$
- Link between them: $\underline{\underline{\boldsymbol{G}}}_{\mathrm{MMSE}} = \underline{\underline{\boldsymbol{L}}}_{p}\underline{\underline{\boldsymbol{D}}}_{LE}$
- Therefore $\underline{\widetilde{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{L}}}_{p}\underline{\widehat{\boldsymbol{B}}}$
- Strictly equivalent up to a proper linear post-processing

Limitation of the previous equalizers

- Both uses the circularization of the channel:
  - Channel matrix diagonal in the FD
  - But time-averaged auto-correlation matrix NOT diagonal
    - Still have an important computational complexity
- Does NOT exploit the circularization of the signal

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A new MMSE-FDE

- Also a MMSE - Frequency Domain Equalizer
- We use the Fractionally-Spaced Representation
- We achieve the same performance than the previous MMSE-FDEs
  - No approximation is made
- ... but with a significantly lower complexity
  - "One-tap" MMSE-FDE
  - by exploiting the circular properties of the signal

Circularization of the signal

- Equivalence between the circular convolution and the linear convolution
- Over a finite-time observation, equivalence between this circular convolution and a linear convolution of $\underline{\boldsymbol{h}}$ and a periodic version of $\underline{\boldsymbol{s}}$
- By considering this periodic version of $\underline{\boldsymbol{s}}$, the time-averaged auto-correlation function of $\underline{\boldsymbol{s}}$ is periodic

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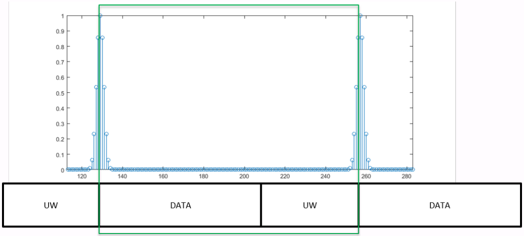

Figure: Periodization of the time-averaged auto-correlation function

Derivation of the equalizer

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- Equalizer given by:

$$\underline{\underline{\boldsymbol{J}}}_{\mathrm{MMSE}} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^{H}(\underline{\underline{\boldsymbol{H}}\boldsymbol{R}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^{H} + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{kN})^{-1} \tag{10}$$

Derivation of the equalizer

- Use the FS representation of the signal as [PV06]
- Considers only the channel contribution as [VT+09]
- Received signal in the FD: $\underline{\boldsymbol{R}} = \underline{\underline{\boldsymbol{H}}}\underline{\boldsymbol{S}} + \underline{\boldsymbol{W}}$
- Equalizer given by:

$$\underline{\underline{\boldsymbol{J}}}_{\mathrm{MMSE}} = \underline{\underline{\boldsymbol{R}}}_{SS}\underline{\underline{\boldsymbol{H}}}^{H}(\underline{\underline{\boldsymbol{H}}}\underline{\underline{\boldsymbol{R}}}_{SS}\underline{\underline{\boldsymbol{H}}}^{H} + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{kN})^{-1} \tag{10}$$

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  - $\underline{\underline{\boldsymbol{r}}}_{ss}$ is circulant
  - $\underline{\underline{\boldsymbol{R}}}_{ss}$ is diagonal, by DFT properties (as $\underline{\underline{\boldsymbol{H}}}$)

$$\underline{\underline{\boldsymbol{r}}}_{ss} = \begin{bmatrix} r_{ss}(0) & r_{ss}^*(1) & r_{ss}^*(2) & \dots & r_{ss}^*(kN-1) \\ r_{ss}(1) & r_{ss}(0) & r_{ss}^*(1) & \dots & r_{ss}^*(kN-2) \\ r_{ss}(2) & r_{ss}(1) & r_{ss}(0) & \dots & r_{ss}^*(kN-3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{ss}(kN-1) & r_{ss}(kN-2) & r_{ss}(kN-3) & \dots & r_{ss}(0) \end{bmatrix} \tag{11}$$

$$= \begin{bmatrix} r_{ss}(0) & r_{ss}(kN-1) & r_{ss}(kN-2) & \dots & r_{ss}(1) \\ r_{ss}(1) & r_{ss}(0) & r_{ss}(kN-1) & \dots & r_{ss}(2) \\ r_{ss}(2) & r_{ss}(1) & r_{ss}(0) & \dots & r_{ss}(3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{ss}(kN-1) & r_{ss}(kN-2) & r_{ss}(kN-3) & \dots & r_{ss}(0) \end{bmatrix} \tag{12}$$

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Structure of the equalizer

- $\underline{\underline{\boldsymbol{J}}}_{\text{MMSE}}$ is a diagonal matrix
- "One-tap" MMSE-FDE:

$$J[l] = \frac{R_{\mathrm{SS}}[l]H^*[l]}{R_{\mathrm{SS}}[l]|H[l]|^2 + \sigma_n^2} \tag{13}$$

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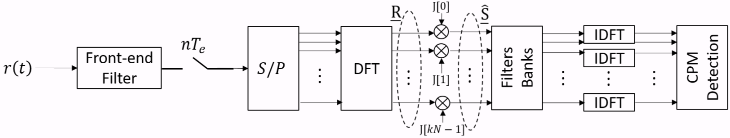
- Use of a conventional CPM Detector
- Has a low-complexity structure *without* any approximation



Difference with the Channel MMSE-FDE [VT+09]

- Difference in the representation of the received signal
- Using the polyphase representation, the time-domain auto-correlation matrix of the over-sampled complex envelope does not have anymore a circular structure

$$\underline{\underline{r}}_{\mathbf{ss,p}} \tag{14}$$

$$= \mathbb{E}[\underline{s}_p \underline{s}_p^H] \tag{15}$$

$$= \begin{bmatrix} r_{\mathbf{ss}}(0) & r^*_{\mathbf{ss}}(2) & r^*_{\mathbf{ss}}(4) & \dots & r^*_{\mathbf{ss}}(2N-2) & r^*_{\mathbf{ss}}(1) & r^*_{\mathbf{ss}}(3) & \dots & r^*_{\mathbf{ss}}(2N-1) \\ r_{\mathbf{ss}}(2) & r_{\mathbf{ss}}(0) & r^*_{\mathbf{ss}}(2) & \dots & r^*_{\mathbf{ss}}(2N-4) & r_{\mathbf{ss}}(3) & r^*_{\mathbf{ss}}(1) & \dots & r^*_{\mathbf{ss}}(2N-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{16}$$

- Its FD counterpart is therefore not diagonal
- Also, the Channel matrix in the FD ($\underline{\underline{\boldsymbol{H}}}_p$) is not a diagonal in the Polyphase Rep.

Summary and Computational complexity comparison

Notation	Reference	Equalizer	Signal Rep.
LD-FS-MMSE-FDE	[PV06]	$\widehat{\underline{\underline{\boldsymbol{B}}}} = \underline{\underline{\boldsymbol{J}\boldsymbol{M}}}^H[\underline{\underline{\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{M}}}^H + \sigma_n^2 I_{2N}]^{-1}\underline{\boldsymbol{R}}$	FS
PP-MMSE-FDE	[VT+09]	$\widehat{\underline{\boldsymbol{S}}}_p = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}}\underline{\underline{\boldsymbol{H}}}_p^H[\underline{\underline{\boldsymbol{H}}}_p\underline{\underline{\boldsymbol{R}}}_{\mathrm{SS,p}}\underline{\underline{\boldsymbol{H}}}_p^H + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{2N}]^{-1}\underline{\boldsymbol{R}}_p$	Polyphase
LD-PP-MMSE-FDE	[Cha+17]	$\widehat{\underline{\boldsymbol{B}}} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{BB}}\underline{\underline{\boldsymbol{P}}}^H[\underline{\underline{\boldsymbol{P}\boldsymbol{R}}}_{\mathrm{BB}}\underline{\underline{\boldsymbol{P}}}^H + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{2N}]^{-1}\underline{\boldsymbol{R}}_p$	Polyphase
FS-MMSE-FDE	[Cha+18]	$\widehat{\underline{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^H[\underline{\underline{\boldsymbol{H}\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^H + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{2N}]^{-1}\underline{\boldsymbol{R}}$	FS

Receiver Type	FFTs and IFFTs	Equalizer Calc.	Equalization
Linear MMSE-TDE [PV06]	0	$+8N^3$	$+\mathcal{O}(PN^3)$
LD-FS-MMSE-FDE [PV06]	$2N\log(2N) + PN\log(N)$	$+8N^3$	$+\mathcal{O}(PN^3)$
PP-MMSE-FDE [VT+09]	$2N\log(N) + PN\log(N)$	$+8N^3$	$+\mathcal{O}(PN^3)$
Approx. PP-MMSE-FDE [VT+09]	$2N\log(N) + PN\log(N)$	$+\mathcal{O}(2N)$	$+\mathcal{O}(PN)$
FS-MMSE-FDE	$2N\log(2N) + PN\log(N)$	$+\mathcal{O}(2N)$	$+\mathcal{O}(PN)$

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FS-MMSE-FDE	[Cha+18]	$\underline{\widehat{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^H[\underline{\underline{\boldsymbol{H}\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}^H + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{2N}]^{-1}\underline{\boldsymbol{R}}$	FS

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Simulation Results (1/2)

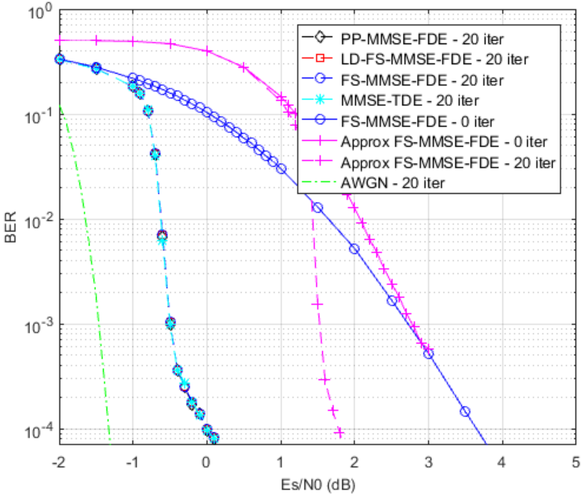

Figure: BER over an aeronautical channel

Simulation Results (2/2)

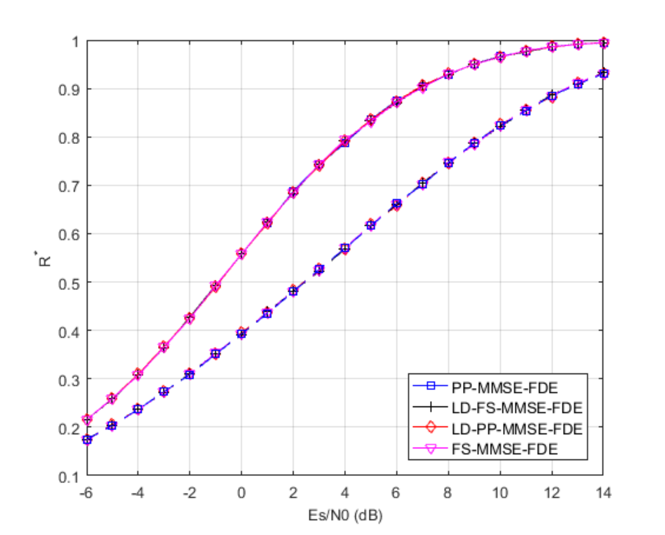

Figure: Maximum achievable coding rate for the different MMSE-FDE over a generic frequency-selective channel

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An extension to Time-Varying Channels: Motivation

- CPM transmission over Time-Variant (TV) channels

$$r[l] = r\left(\frac{lT}{k}\right) = \sum_m s[m]h[l; l-m] + w[l] \qquad (17)$$

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  - Complexity growing with the channel span
  - Capitalizes on the Laurent Decomposition

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- State of the Art [Dar+16]
  - MMSE Time-Domain Equalizer
  - Complexity growing with the channel span
  - Capitalizes on the Laurent Decomposition
- Main Issue for MMSE-FDE
  - FD channel matrix is no more diagonal
  - Requires higher computation (inversion of full matrix)

Band MMSE-FDE: Motivation

- Main idea: exploit the band structure of the channel matrix in the FD ([RBL06])

$$\underline{\underline{\boldsymbol{H}}} \approx \underline{\underline{\boldsymbol{H}}}_Q = \underline{\underline{\boldsymbol{B}}}^{(Q)} \circ \underline{\underline{\boldsymbol{H}}} \qquad (18)$$

$\underline{\underline{\boldsymbol{B}}}^{(Q)}$ matrix with 1's only on Q sub-diagonals
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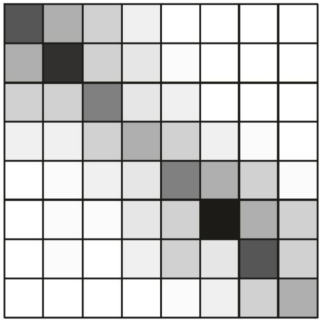

Figure: FD channel matrix from [RBL06]

Band MMSE-FDE: Derivation & Procedure

- Linear block MMSE equalizer given by:

$$\underline{\underline{\boldsymbol{J}}}_{\mathrm{MMSE},Q} = \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}} \underline{\underline{\boldsymbol{H}}}_{Q}^{H} \left(\underline{\underline{\boldsymbol{H}}}_{Q} \underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}} \underline{\underline{\boldsymbol{H}}}_{Q}^{H} + \sigma_n^2 \underline{\underline{\boldsymbol{I}}}_{2N} \right)^{-1} \qquad (19)$$

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- Capitalization on the band structure of $\underline{\underline{\boldsymbol{J}}}_{\mathrm{MMSE},Q}$
- Procedure:
  - Compute the band matrix $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{H}}}_{Q}\underline{\underline{\boldsymbol{R}}}_{\mathrm{SS}}\underline{\underline{\boldsymbol{H}}}_{Q}^{H} + \sigma_n^2\underline{\underline{\boldsymbol{I}}}_{kN}$;
  - Compute the LDL decomposition of $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{L}\boldsymbol{D}\boldsymbol{L}}}^{H}$ where $\underline{\underline{\boldsymbol{L}}}$ is a lower triangular matrix and $\underline{\underline{\boldsymbol{D}}}$ a diagonal matrix following [RBL05] ;
  - Solve the triangular system $\underline{\underline{\boldsymbol{L}}}\underline{\boldsymbol{f}} = \underline{\underline{\boldsymbol{R}}}$;
  - Solve the diagonal system $\underline{\underline{\boldsymbol{D}}}\underline{\boldsymbol{g}} = \underline{\boldsymbol{f}}$;
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  - Solve the triangular system $\underline{\underline{\boldsymbol{L}}}^H\underline{\boldsymbol{d}} = \underline{\boldsymbol{g}}$
- Computational complexity: $O\big(kN(2Q^2 + Q + \log(kN)\big)$

Simulation Results (1/3)

- "En Route" Scenario:
  - Aeronautical channel by satellite
  - C Band
  - Power ratio between the two paths: $C/M = 5dB$
  - Doppler Spread of $500Hz$

Simulation Results (2/3)

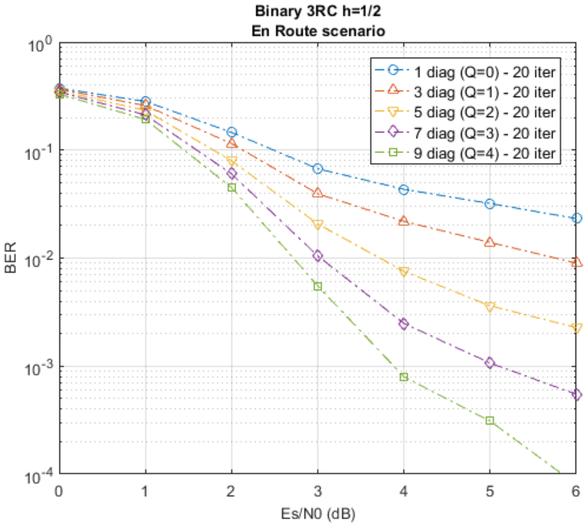

Figure: Influence of the parameter Q

Simulation Results (3/3)

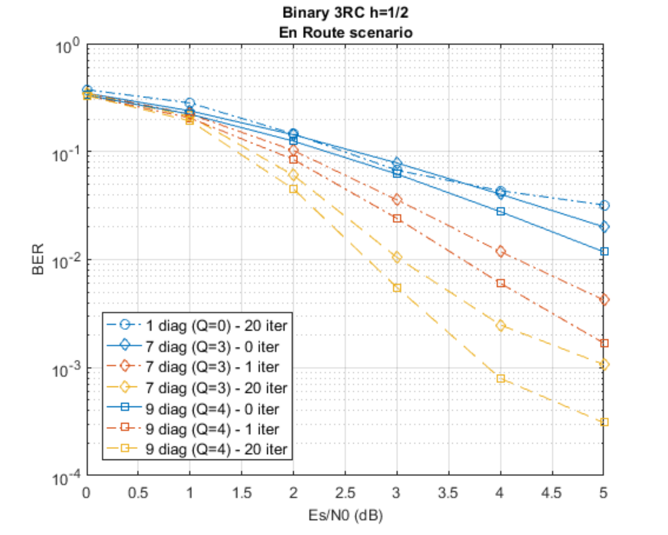

Figure: Influence of the parameter Q and of the number of iteration

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Motivation

- Previous work -> perfect synchronization and perfect channel knowledge
- CFO, phase and timing recovery
  - most of those methods for CPM transmission over AWGN channels
  - CRB derived in case of AWGN channels [HP13]
- Case of interest: transmission over TIV and TV channels

Contributions

- Channel Estimation
  - Case of TIV channels
  - Case of TV channels (using BEM)
- Joint Carrier Frequency Offset and Channel Estimation
  - Case of TIV channels
  - Case of TV channels
  - Derivation of the Cramér Rao Bound

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System Model

- Received signal:

$$\begin{bmatrix} r[0] \\ r[1] \\ \vdots \\ \vdots \\ r[kJ-1] \end{bmatrix} = \begin{bmatrix} s[0] & s[-1] & \dots & s[-L_h+1] \\ s[1] & s[0] & s[-1] & \dots \\ s[L_h-1] & \dots & s[1] & s[0] \\ \ddots & \ddots & \ddots & \ddots \\ s[kJ-1] & \dots & s[kJ-L_h+1] & s[kJ-L_h] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ \vdots \\ h[L_h-1] \end{bmatrix} + \underline{\boldsymbol{w}} \quad (20)$$

$$= \underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}} + \underline{\boldsymbol{w}} \quad (21)$$

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$$= \underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}} + \underline{\boldsymbol{w}} \tag{21}$$

- Difference with linear modulations:
  - We do not estimate a discrete equivalent channel (at baud rate)
  - We estimate an over-sampled channel filtered by a LPF at the receiver

Channel to estimate

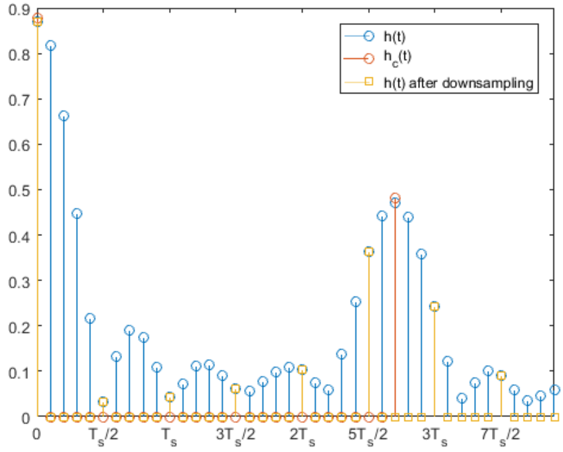

Figure: Finite impulse responses of $h_c(t)$ and $h(t)$

Channel Estimation

- Least Squares Channel estimate:

$$\widehat{\underline{\boldsymbol{h}}}_{\mathrm{LS}} = (\underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{s}}})^{-1} \underline{\underline{\boldsymbol{s}}}^H \underline{\boldsymbol{r}} \tag{22}$$

- Works at kR_s ($k \geq 2$)
- Can be improved by using an *a priori* knowledge of the delays (aeronautical channel by satellite)
- Can be extended to TV Channels using Basis Expansion Models (*BEM*)

Simulation Results

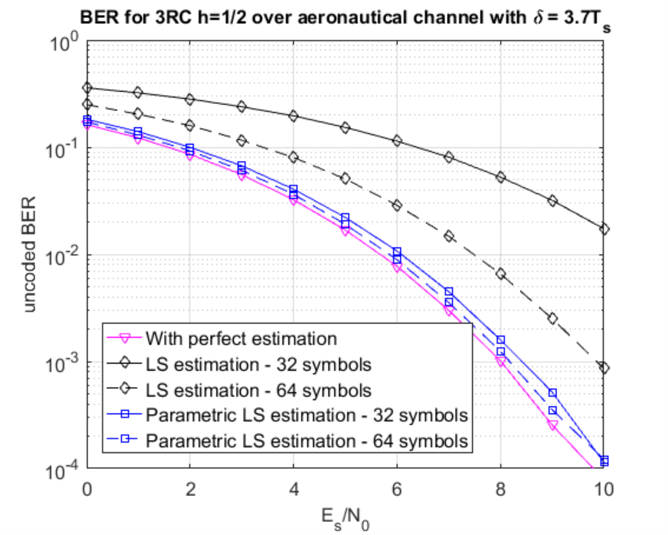

Figure: Uncoded BER over the aeronautical channel with channel estimation

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System Model

- Introduction of a Carrier Frequency Offset:

$$\underline{r} = \underline{\underline{\Gamma}}(f)\underline{\underline{s}}\underline{h} + \underline{w} \tag{23}$$

with

$$\underline{\underline{\Gamma}}(f) = \mathrm{diag}\{1, e^{j2\pi f T_e}, e^{j2\pi f 2T_e}, \dots, e^{j2\pi f (kJ-1)T_e}\}$$

$$\underline{\underline{s}} = \begin{bmatrix} s[0] & 0 & \dots & 0 \\ s[1] & s[0] & 0 & \vdots \\ s[L_h-1] & \dots & s[1] & s[0] \\ \ddots & \ddots & \ddots & \ddots \\ s[kJ-1] & \dots & s[kJ-L_h+1] & s[kJ-L_h] \end{bmatrix}$$

$$\underline{h} = \begin{bmatrix} h[0] & h[1] & \dots & \dots & h[L_h-1] \end{bmatrix}^T$$

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Joint ML Estimation of $\underline{\boldsymbol{h}}$ and the CFO f

- [MM00] for linear modulations
- Log-likelihood function for the parameters $\underline{\boldsymbol{h}}$ and f is:

$$\Delta(\underline{\widetilde{\boldsymbol{h}}}, \widetilde{f}) = -\frac{1}{\sigma_n^2}[\underline{\boldsymbol{r}} - \underline{\underline{\boldsymbol{\Gamma}}}(\widetilde{f})\underline{\underline{\boldsymbol{s}}}\underline{\widetilde{\boldsymbol{h}}}]^H[\underline{\boldsymbol{r}} - \underline{\underline{\boldsymbol{\Gamma}}}(\widetilde{f})\underline{\underline{\boldsymbol{s}}}\underline{\widetilde{\boldsymbol{h}}}] \qquad (24)$$

- Maximize Δ over $\underline{\widetilde{\boldsymbol{h}}}$ and $\widetilde{f}$
- Estimate of $\underline{\boldsymbol{h}}$ for a given $\widetilde{f}$ is

$$\underline{\widehat{\boldsymbol{h}}}(\widetilde{f}) = (\underline{\underline{\boldsymbol{s}}}^H\underline{\underline{\boldsymbol{s}}})^{-1}\underline{\underline{\boldsymbol{s}}}^H\underline{\underline{\boldsymbol{\Gamma}}}^H(\widetilde{f})\underline{\boldsymbol{r}} \qquad (25)$$

- We use this estimate in Eq.(24) to estimate f

Procedure

- Compute $\widehat{f}$ as follows:

$$\widehat{f} = \arg\max_{\widetilde{f}} \Big(-\rho(0) + 2\Re\{ \sum_{m=0}^{kJ-1} \rho(m) e^{-j2\pi \widetilde{f} m} \} \Big) \quad (26)$$

$$\text{where } \rho(m) = \sum_{l=0}^{kJ-1} [\underline{\underline{\boldsymbol{B}}}]_{l-m,m} r[l] r^*[l-m] \quad (27)$$

$$\text{and } \underline{\underline{\boldsymbol{B}}} \triangleq \underline{\underline{\boldsymbol{s}}}(\underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{s}}})^{-1} \underline{\underline{\boldsymbol{s}}}^H \quad (28)$$

- Compute $\widehat{\underline{\boldsymbol{h}}}$ using the LS estimate on the counter-rotated received samples using $\widehat{\underline{\boldsymbol{h}}}(\widetilde{f}) = (\underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{s}}})^{-1} \underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{\Gamma}}}^H(\widetilde{f}) \underline{\boldsymbol{r}}$.

Simulation Results (1/2)

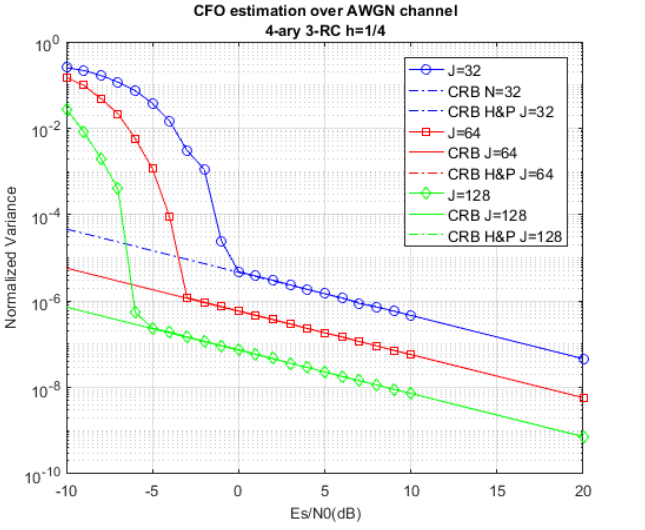

Figure: Carrier Recovery for 4-ary CPM over AWGN channel

Simulation Results (2/2)

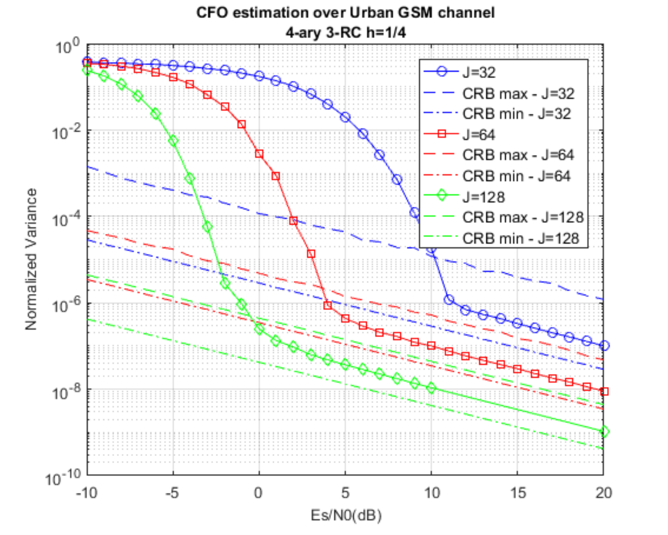

Figure: Carrier Recovery for Quaternary 3-RC h=1/4 over Urban GSM channel

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Conclusions

- Derivation a low-complexity MMSE-FDE for CPM
  - Exact for TIV channels
  - Approximate for TV channels

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  - In case of TV channel (using BEM)
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Conclusions

- Derivation a low-complexity MMSE-FDE for CPM
  - Exact for TIV channels
  - Approximate for TV channels
- Derivation of a LS Channel Estimation
  - In case of TIV channel
  - In case of TV channel (using BEM)
  - Parametric Estimation with an *a priori* knowledge on the delays
- Derivation of a joint ML Channel and CFO Estimator
  - For both TIV and TV channels
  - Reaches the CRB (also derived) asymptotically

Perspectives

- Non-linear equalization scheme (DFE or Turbo)

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- Non-linear equalization scheme (DFE or Turbo)
- Receiver Windowing for Band MMSE-FDE for CPM transmissions over doubly-selective channels
- Preamble design
- Timing Recovery
- Optimal BEM model for the aeronautical channel

List of publications

Journal Paper:

- "A New Exact Low-Complexity MMSE Equalizer for Continuous Phase Modulation", *IEEE Communications Letters*, 2018

International conference papers:

- "Doubly-Selective Channel Estimation for Continuous Phase Modulation", *IEEE Int. Military Communications Conference (MILCOM)*, Los Angeles (CA), U.S.A, 2018
- "A Frequency-Domain Band-MMSE Equalizer for Continuous Phase Modulation over Frequency-Selective Time-Varying Channels", *European Signal Processing Conference (EUSIPCO)*, Rome, Italy, 2018
- "Channel Estimation and Equalization for CPM with application for aeronautical communications via a satellite link", *IEEE Int. Military Communications Conference (MILCOM)*, Baltimore (MD), U.S.A, 2017
- "Joint Channel and Carrier Frequency Estimation for M-ary CPM over frequency-selective channel using PAM decomposition", *IEEE Int. Conf. Acoust., Speech, and Signal Proc. (ICASSP)*, New Orleans LA, U.S.A, 2017

National conference paper:

- "Sur l'égalisation fréquentielle des modulations à phase continue", *Colloque GRETSI sur le traitement du Signal*, Juan-les-pins, France, 2017

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MMSE-FDE [TS05] (1/2)

Based on an orthogonal basis of the signal space $\{f_k(t)\}$

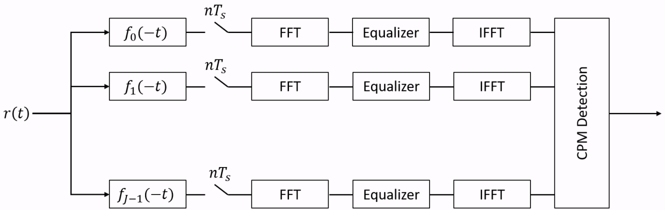

Figure: MMSE-FDE with orthogonal representation from [TS05]

Requires delays multiple of T_s

MMSE-FDE [TS05] (2/2)

Based on the Laurent Decomposition

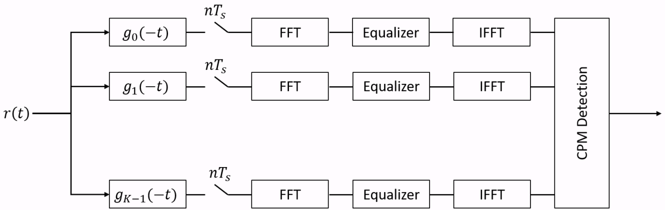

Figure: MMSE-FDE from [TS05]

Does not take into account the auto-correlation of the pseudo-symbols

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Parametric Least Squares Estimation (1/2)

- directly estimate the attenuations $\{h_l\}_l$ and the delays $\{\tau_l\}_l$ of the different path of the channel
- the delays are known in this aeronautical context by GPS and geometrical consideration
- Introduction of the dependency on those delays:

$$\underline{\underline{\boldsymbol{P}}}(\underline{\boldsymbol{\tau}}) = \begin{bmatrix} \Psi(0-\tau_0) & \dots & \Psi(0-\tau_{L_c-1}) \\ \Psi(T_e-\tau_0) & \dots & \Psi(T_e-\tau_{L_c-1}) \\ \vdots & \dots & \vdots \\ \Psi(kNT_e-\tau_0) & \dots & \Psi(kNT_e-\tau_{L_c-1}) \end{bmatrix} \quad (29)$$

and so $\underline{\boldsymbol{h}} = \underline{\underline{\boldsymbol{P}}}(\underline{\boldsymbol{\tau}})[h_0, h_1, \dots, h_{L_c-1}]^T$ (30)

Parametric Least Squares Estimation (2/2)

- New system model:

$$\underline{r} = \underline{\underline{s}}\underline{h} + \underline{w} = \underbrace{\underline{\underline{s}}\,\underline{\underline{P}}(\underline{\tau})}_{=\underline{\underline{s}}(\underline{\tau})}\underline{a} + \underline{w} \tag{31}$$

$$= \underline{\underline{s}}(\underline{\tau})\underline{a} + \underline{w} \tag{32}$$

- LS channel estimation:

$$\widehat{\underline{a}} = (\underline{\underline{s}}(\tau)^H \underline{\underline{s}}(\tau))^{-1} \underline{\underline{s}}(\tau)^H \underline{r} \tag{33}$$

$$\text{and } \widehat{\underline{h}} = \underline{\underline{P}}(\tau)\widehat{\underline{a}} \tag{34}$$

- lower bound of the more generic case where a joint delays and attenuations estimation
- lower complexity and better performance

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System Model

- We assume perfect timing synchronization
- Unknown parameters:
  - Propagation Channel
  - Carrier Frequency Offset
  - Initial Phase
- Doubly-selective channel:

$$\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{h}}}\underline{\boldsymbol{s}} + \underline{\boldsymbol{w}} \tag{35}$$

with $\underline{\boldsymbol{r}} = [r[0], r[1], \ldots, r[kN-1]]^T$

$\underline{\boldsymbol{s}} = [s[0], s[1], \ldots, s[kN-1]]^T$

and $\underline{\boldsymbol{w}} = [w[0], w[1], \ldots, w[kN-1]]^T$

with

$$\underline{\underline{\boldsymbol{h}}} = \begin{bmatrix} h[0,0] & 0 & \ldots & \ldots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ h[L-1, L-1] & & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & 0 \\ 0 & \ldots & h[kN-1, L-1] & \ldots & h[kN-1, 0] \end{bmatrix} \tag{36}$$

Basis Expansion Model (BEM)

- Complex attenuation of the l^{th} path:

$$\underline{\boldsymbol{h}}_l = \Big[h[0,l], h[1,l], \ldots, h[kN-1,l]\Big]^T \tag{37}$$

$$= \underbrace{[\underline{\zeta}_0, \underline{\zeta}_1, \ldots, \underline{\zeta}_{P-1}]}_{\underline{\underline{\zeta}}} \underbrace{[\eta_{l,0}, \eta_{l,1}, \ldots, \eta_{l,P-1}]^T}_{=\underline{\boldsymbol{\eta}}_l} \tag{38}$$

$$= \sum_{p=0}^{P-1} \eta_{l,p} \underline{\zeta}_p \tag{39}$$

- $\{\underline{\zeta}_p\}$ is a basis of the attenuation's vectors space
- This basis is deterministic
- Only have to estimate the $P(L-1)$ coefficients $\{\eta_{l,p}\}$ $(P << kN-1)$

Matrix-wise representation of the received signal

- Channel Matrix:

$$\underline{\underline{\boldsymbol{h}}} = \sum_{l=0}^{L-1} \underline{\underline{\boldsymbol{Z}}}_l \mathrm{diag}(\underline{\boldsymbol{h}}_l) \tag{40}$$

$$= \sum_l \sum_p \eta_{l,p} \underbrace{\mathrm{diag}(\underline{\zeta}_p)\underline{\underline{\boldsymbol{Z}}}_l}_{=\underline{\underline{\boldsymbol{\Omega}}}_{l,p}} = \sum_l \sum_p \eta_{l,p} \underline{\underline{\boldsymbol{\Omega}}}_{l,p} \tag{41}$$

$$= \underline{\underline{\boldsymbol{\Omega}}}(\underline{\boldsymbol{\eta}} \otimes \underline{\underline{\boldsymbol{I}}}_{kN}) \tag{42}$$

- Received signal:

$$\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{\Omega}}}(\underline{\boldsymbol{\eta}} \otimes \underline{\underline{\boldsymbol{I}}}_{kN})\underline{\boldsymbol{s}} + \underline{\boldsymbol{w}} \tag{43}$$

$$= \underline{\underline{\boldsymbol{\Omega}}}(\underline{\underline{\boldsymbol{I}}}_{LP} \otimes \underline{\boldsymbol{s}})\underline{\boldsymbol{\eta}} + \underline{\boldsymbol{w}} \tag{44}$$

LS Estimation of the BEM parameters

- Similar to the TIV case, let $\underline{\underline{\boldsymbol{s}}} \triangleq \underline{\underline{\boldsymbol{\Omega}}}(\underline{\underline{\boldsymbol{I}}}_{LP} \otimes \underline{\boldsymbol{s}})$
- LS channel estimate:

$$\underline{\widehat{\boldsymbol{\eta}}} = (\underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{s}}})^{-1} \underline{\underline{\boldsymbol{s}}}^H \underline{\boldsymbol{r}} \tag{45}$$

$$\text{and so } \underline{\underline{\widehat{\boldsymbol{h}}}} = \underline{\underline{\boldsymbol{\Omega}}}(\underline{\widehat{\boldsymbol{\eta}}} \otimes \underline{\underline{\boldsymbol{I}}}_{kN}) \tag{46}$$

- Can be performed on circular block based CPM transmission using UW

Simulation Results (1/3)

- Scenario:
  - TV aeronautical channel by satellite
  - C-band
  - $C/M = 5\text{dB}$
  - Doppler Spread of $0.0183R_S$, then $0.0008R_s$

Simulation Results (2/3)

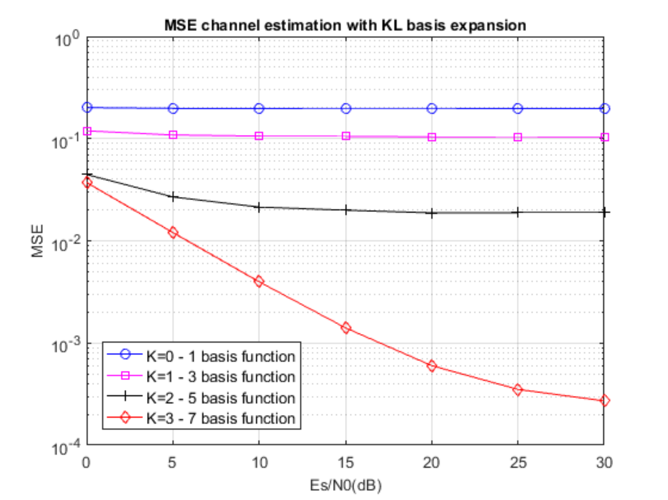

Figure: NMSE over TV channels using KL-BEM

Simulation Results (3/3)

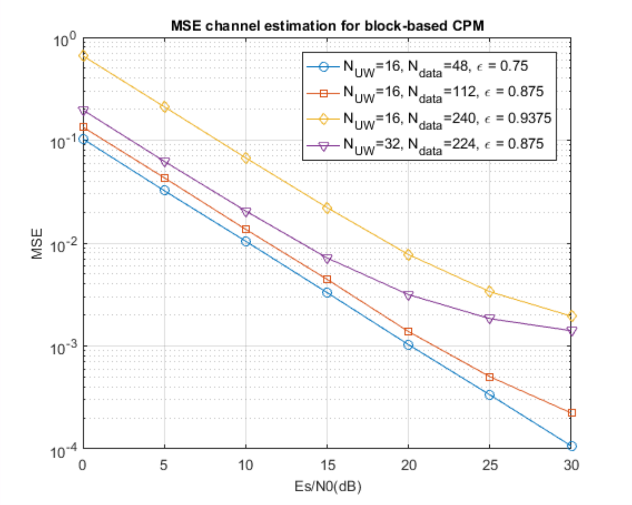

Figure: NMSE over TV channels using KL-BEM for block-based CPM

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Cramér Rao Bound for carrier recovery in case of TIV Channel

- Sys. Mod.: $\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{\Gamma}}}(f)\underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}} + \underline{\boldsymbol{w}}$
- Set of unknown parameters: $u = (\underline{\boldsymbol{h}}_r, \underline{\boldsymbol{h}}_i, f)$
- Fisher Information Matrix:

$$[\underline{\underline{\boldsymbol{F}}}]_{i,j} = -E\left[\frac{\partial^2 \ln \Delta(\underline{\boldsymbol{r}}, u)}{\partial u(i)\partial u(j)}\right]$$

- $\mathrm{CRB}(f) = \frac{\sigma_n^2}{2\underline{\boldsymbol{y}}^H(\underline{\underline{\boldsymbol{I}}}_{kJ} - \underline{\underline{\boldsymbol{B}}})\underline{\boldsymbol{y}}}$

$$\text{with } \underline{\boldsymbol{y}} = 2\pi\underline{\underline{\boldsymbol{M}}}\underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}}$$
$$\text{and } \underline{\underline{\boldsymbol{M}}} = \mathrm{diag}(0, 1, \ldots, kJ-1)$$

- AWGN Case:

$$\mathrm{CRB}_{\mathrm{AWGN}}(f) = \frac{\sigma_n^2}{2\underline{\boldsymbol{y}}^H(\underline{\underline{\boldsymbol{I}}}_{kJ} - \underline{\underline{\boldsymbol{B}}})\underline{\boldsymbol{y}}} \approx \frac{3}{2\pi^2 J^3}(SNR)^{-1}$$

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Pragmatic CPM

- cf Thesis "Nouvelle forme d'onde et récepteur avancé pour la télémesure des futurs lanceurs", by C.-U. Piat-Durozoi